# Design Tolerance Optimization using LS-OPT

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## 1 Introduction

In design optimization, it is typical to minimize an objective function, such as the cost of the design, while constraining other responses. As a result, the final design often lies at the limit of permissible behavior. However, a slight perturbation of the design or the loading conditions can lead to an inadmissible response. In addition, the objective function may be highly sensitive to perturbations leading to a much worse value than the expected optimum values. To avoid these issues, reliability-based design optimization (RBDO) and robust optimization are often performed [1,2]. Tolerance values play an important role in determining the effect of uncertainties on the optimal design [3-5]. The tolerance intervals define the variable probability density function (PDF) bounds and, therefore, affect the failure probability. In the absence of complete probability density information, they facilitate the search for a worst case design; an optimal design with zero failure within the tolerance intervals can be determined.

In this work, LS-OPT is used to perform tolerance-based design optimization using a multi-level scheme [6]. The outer level consists of an optimization problem that maximizes the tolerance value such that there is no failure within the tolerance interval (i.e. neligible probability of failure). The nominal design parameters and their tolerance values are the optimization variables for this level; thus, each sample in the outer level uniquely defines the nominal values and bounds of the variable probability density functions (PDFs). The probability of failure for each sample is determined using an inner level Monte Carlo analysis. This value is then extracted as an outer level response and is constrained to be close to zero during the optimization. The outer level optimization is formulated such that a balance between robustness and performance is acheved.

For problems with expensive analysis, e.g. crash anaysis, a two step approach is presented. For such problems, the first step involves the construction of high fidelity global metamodels for the system responses and saving their formulae to a file. In the second step, the previously constructed metamodels are used to replace the actual expensive function analysis. In other words, once the global metamodels are constructed, the multi-level tolerance scheme does not need any expensive function evaluation.

The tolerance optimization methodology is applied to optimize the thickness design parameters for a Chevrolet truck impact problem as well as the associated tolerances. The details of the LS-OPT setup are illustrated through this design problem. The newly developed features and entities of LS-OPT used in this work include multi-level optimization using LS-OPT stage, transfer variable usage, exporting metamodel formulae to a file and parametrization of LS-OPT attributes such as distrbution properties. A multi-objective tolerance-based optimization is performed to obtain a tradeoff.

In the following sections, the tolerance optimization method is explained in greater detail along with some results. In Section 2, the tolerance optimization formulation is provided. Section 3 involves the the applicaiton of the proposed methodology to a Chevrolet pickup truck along with the steps required for setting up the problem in LS-OPT. In Section 4, results for the truck optimization problem are presented. Finally, Section 5 provides a summary of the work and some future work that will help in further streamlining the setup in LS-OPT.

## 2 Tolerance Optimization Methodology

This section presents the formulation of the tolerance optimization [3-5] problem solved in this work. The methodology used in this work is summarized using Figure 1. In general, tolerance optimization involves a large number of function evaluations. Therefore, to avoid high computation cost for problems with expensive analysis, the methodology used in this work consists of two steps. Metamodels for the responses are constructed in the first step and these computationally inexpensive approximations are then used for the tolerance optimization, which is performed in the second step.

First, in step 1, a zero tolerance problem is solved as a deterministic single iteration optimization using LS-OPT. The optimization problem can be formulated as:

$$
\begin{aligned}
&\min_{\overline{x}} && f_1(\overline{x}) \\
&s.t. && g_i(\overline{x}) - 1 \le 0, \ i = 1,2,3 \\
&with && \overline{x}_l \le \overline{x} \le \overline{x}_u
\end{aligned}
\tag{1}
$$

where the optimization variables $\overline{x}$ are nominal variables. It should be noted that the solution of the above optimization formulation may not be robust. However, the above problem is solved to achieve two objectives - to obtain a target value of $f_1$ for step 2, where a constraint is applied on $f_1$, and to obtain high fidelity metamodel approximations of the responses that are used to replace the expensive analysis in step 2 tolerance optimization. It is important to obtain the response approximations as replacements of the expensive analysis, as tolerance optimization involves a high number of function evaluations.

In step 2, the tolerance optimization is solved as a single or multi-objective problem. This work consists of a multi-objective optimization with two objective functions. One of the two objectives is a tolerance scale factor, which is the ratio of the tolerance and nominal values, and the second objective is the nominal value of the step 1 objective function $f_1$. It should be noted that $f_1$ can be a vector of multiple objective functions, but is treated as a scalar here for simplicity. The optimization in step 2 is set up as a two-level problem in LS-OPT. The nominal design variables and the tolerance value are optimized in the outer level while the inner level computes the probability of failure at any design alternative. The failure probabilities in the inner level are calculated based on the previously constructed metamodel approximations.

The outer level optimization is:

$$
\begin{aligned}
&\max_{\mathbf{t},\overline{\mathbf{x}}} && \{\mathbf{t}, -f_1\} \\
&s.t. && \mathbf{g}(\mathbf{x}) \le \mathbf{0} && \forall && \mathbf{x} \in [\overline{\mathbf{x}} - \boldsymbol{\delta}_t, \overline{\mathbf{x}} + \boldsymbol{\delta}_t] \\
& && f_1(\mathbf{x}) \le \eta f_1^* && \forall && \mathbf{x} \in [\overline{\mathbf{x}} - \boldsymbol{\delta}_t, \overline{\mathbf{x}} + \boldsymbol{\delta}_t]
\end{aligned}
\tag{2}
$$

where $t$ is the vector of relative tolerance values or the tolerance scale factors, $\boldsymbol{\delta}_t$ is the vector of absolute tolerance values, $f_1^*$ is the optimal objective function value for step 1 solution and $\eta$ is a factor greater than 1. It should be noted that the absence of failure within a large tolerance interval implies high robustness of the solution, but a very large tolerance will also lead to loss of performance. Therefore, it is important to include the original objective function $f_1$ as a constraint while maximizing the tolerance. A design obtained in this manner strikes a balance between robustness and nominal performance. Figure 1 shows the general overview of the multilevel tolerance-based multiobejctive optimization. Equation (2) can also be written as:

$$
\begin{aligned}
&\max_{\mathbf{t},\overline{\mathbf{x}}} && \{\mathbf{t}, -f_1\} \\
&s.t. && \mathbf{P}_f(\overline{\mathbf{x}}, \mathbf{t}) \le \mathbf{P}_{target} \\
&where && \mathbf{P}_f = \{P(g_1(\mathbf{x} \mid \overline{\mathbf{x}}, \mathbf{t}) > 0), \cdots, P(g_p(\mathbf{x} \mid \overline{\mathbf{x}}, \mathbf{t}) > 0), P(f_1(\mathbf{x} \mid \overline{\mathbf{x}}, \mathbf{t}) > \eta f_1^*)\} \text{ and } \mathbf{P}_{target} \approx \mathbf{0}.
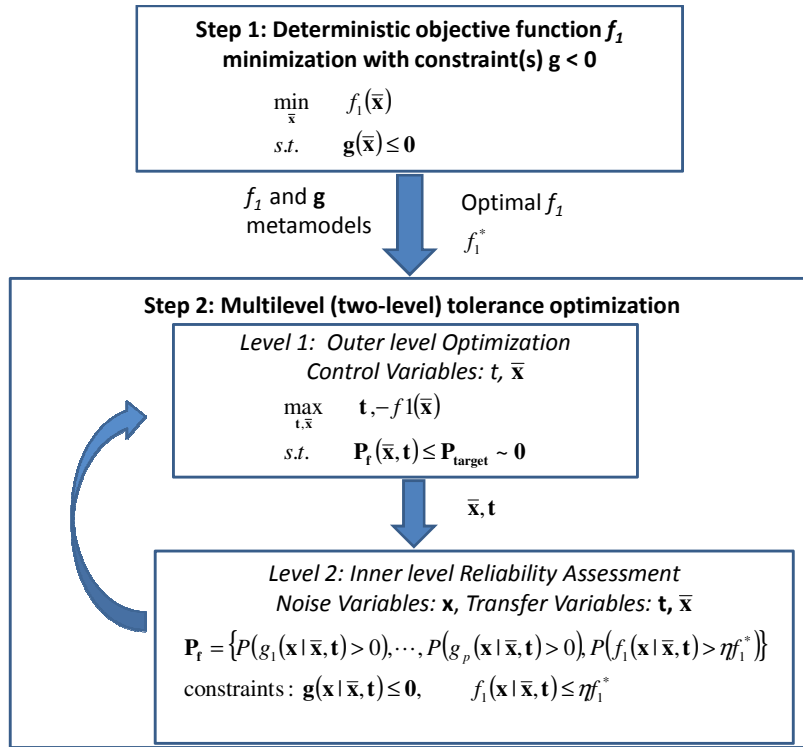\end{aligned}
\tag{3}
$$

<div align="center">

**Step 1: Deterministic objective function $f_1$ minimization with constraint(s) g < 0**

$$\min_{\overline{\mathbf{x}}} \quad f_1(\overline{\mathbf{x}})$$
$$s.t. \quad \mathbf{g}(\overline{\mathbf{x}}) \leq \mathbf{0}$$

</div>

$f_1$ and **g** metamodels

Optimal $f_1$
$f_1^*$

**Step 2: Multilevel (two-level) tolerance optimization**

*Level 1: Outer level Optimization*
*Control Variables: t, $\overline{\mathbf{x}}$*

$$\max_{\mathbf{t},\overline{\mathbf{x}}} \quad \mathbf{t}, -f1(\overline{\mathbf{x}})$$
$$s.t. \quad \mathbf{P}_f(\overline{\mathbf{x}},\mathbf{t}) \leq \mathbf{P}_{\text{target}} \sim \mathbf{0}$$

$\overline{\mathbf{x}}, \mathbf{t}$

*Level 2: Inner level Reliability Assessment*
*Noise Variables: **x**, Transfer Variables: **t**, $\overline{\mathbf{x}}$*

$$\mathbf{P}_f = \left\{ P(g_1(\mathbf{x}|\overline{\mathbf{x}},\mathbf{t}) > 0), \cdots, P(g_p(\mathbf{x}|\overline{\mathbf{x}},\mathbf{t}) > 0), P(f_1(\mathbf{x}|\overline{\mathbf{x}},\mathbf{t}) > \eta f_1^*) \right\}$$
$$\text{constraints}: \mathbf{g}(\mathbf{x}|\overline{\mathbf{x}},\mathbf{t}) \leq \mathbf{0}, \quad f_1(\mathbf{x}|\overline{\mathbf{x}},\mathbf{t}) \leq \eta f_1^*$$

*FIGURE 1 Tolerance Optimization Summary*

## 3  Tolerance Optimization Using LS-OPT - Design Problem and Setup

The design problem discussed in this paper consists of a multi-objective design optimization formulated to optimize the finite element (FE) model of the Chevrolet C2500 pickup truck developed by National Crash Analysis Center (NCAC) [7]. The goal of the optimization process was to obtain a lighter and robust design without compromising the crashworthiness characteristics of the vehicle model given by the intrusion distance and crash pulse responses. The thickness of a few parts selected based on their contribution to the overall crash energy absorption were treated as the design variables. A total of nine parts were selected and due to symmetry in the design, the number of design variables was reduced to six. Figure 2 shows the FE model of the Chevrolet C2500 pickup truck and the design parts selected for the optimization.
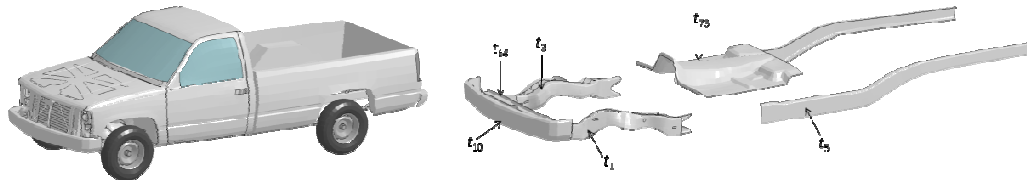


*FIGURE 2  FE model of Chevrolet C2500 pickup truck (left) and the parts selected for optimization (right).*

The tolerance optimization methodology explained in section 2 was applied to the Chevrolet C2500 pickup truck. The following discussion pertains to the setup of the tolerance optimization of the vehicle model using LS-OPT. The optimization is performed in two steps. The first step consists of a deterministic optimization and the tolerance optimization is performed in the second step. The second step further consists of a two-level (multi-level) setup as explained below.

<u>Step 1:</u> The determinitic zero tolerance optimization problem required for step 1 was formulated as:

$$\min_{\overline{x}} \quad scaled\_mass\,(\overline{\boldsymbol{x}})$$

$$s.t. \quad scaled\_stage_i\_pulse(\overline{\boldsymbol{x}}) - 1 \le 0, \ i = 1,2$$
$$scaled\_disp(\overline{\boldsymbol{x}}) - 1 \le 0 \tag{4}$$
$$with \quad \overline{\boldsymbol{x}}_l \le \overline{\boldsymbol{x}} \le \overline{\boldsymbol{x}}_u$$

where $scaled\_mass$ is the total mass of the selected parts normalized by its baseline value, $scaled\_stage_i\_pulse$ are the normalized design constraints given by the crash pulse responses and $scaled\_disp$ is the normalized design constraints given by the the intrution distance. These are required to be less than or at least equal to their baseline values. $\bar{x}$ are the part thickness design variables that need to be between their respective lower and upper bounds.

The optimization problem given by Equation 4 is solved using a metamodel-based single iteration task available in LS-OPT. A relatively high number of samples (1000) were used in this step to obtain high fidelity global metamodel approximations for the responses. This, however, allows one to avoid many more simulations in the potentially expensive second step. The following figure shows the setup required in the LS-OPT tool for solving *Step 1*.
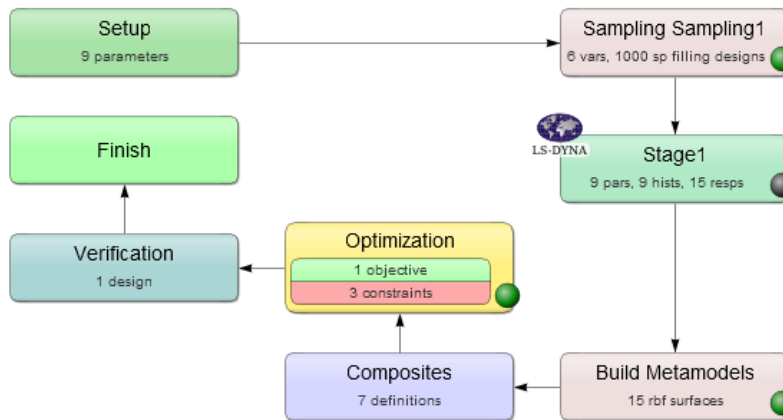


*FIGURE 3 LS-OPT setup for Step 1.*

Radial basis function (RBF) networks were used to formulate the approximate equations for the design responses using a total of 1000 sample points selected using the space filling sampling technique. Through optimization, the total scaled mass of the selected parts was reduced to 0.8323 compared to the baseline value of 1.

The single iteration deterministic optimization in step 1 serves two purposes. First, the deterministic optimal total mass value, after some relaxation, is used as a target to constrain the mass during the tolerance-based optimization in the second step. This ensures robustness of the mass in the final design. In addition, the mathematical expressions of the RBF metamodels constructed in this step are automatically saved into the *DesignFunctions.1* file. These approximations are used to replace the expensive LS-DYNA FE crash analysis during the tolerance optimization in the second step. It should be noted that LS-OPT saves the approximation formulae in Matlab compatible format. In this work, however, the formulae in the *DesignFunctions* file were manually converted to perl compatible format using some simple modifications.

<u>Step 2:</u> This step involves multi-objective optimization with total scaled mass of the design parts and the tolerance scale factor as the objectives. The optimization in step 2 is set up as a two-level (outer and inner) problem in LS-OPT. The outer level consists of a direct simulation-based multiobjective optimization with a tolerance scale factor and the nominal thickness of the design parts as the design

variables. The analysis stage of the outer level consists of an LS-OPT stage that represents the inner level Monte Carlo analysis. Thus, for each outer level sample corresponding to a unique nominal variable value and tolerance value, an inner level Monte Carlo analysis is conducted to compute the corresponding probabilities of failure.

The outer level LS-OPT stage itself represents an inner level LS-OPT setup that contains a user-defined stage with a perl script as the input file. The perl script computes the response values for the truck impact and is based on the step 1 metamodel approximations. An example of a simplified perl script obtained by modifying the *DesignFunctions* file of the step 1 is shown in Figure 4. The script contains 9 parameters that are automatically detected in the inner level setup. However, three of these are dependent on the other variables. The remaining 6 parameters are defined as noise variables.

```perl
#!/usr/bin/perl
open(MASS,">MASS");
  $t1= <<t1>>;
  $t10= <<t10>>;
  $t2= <<t2>>;
  $t3= <<t3>>;
  $t4= <<t4>>;
  $t5= <<t5>>;
  $t6= <<t6>>;
  $t64= <<t64>>;
  $t73= <<t73>>;

$MASS = ((0.045634*((-0.074451 + 0.173058 * ((1.000000*(($t1 - 2.5096)/1.2548)-
0.000424)/0.999262) + 0.142466 * ((1.000000*(($t10 - 2.16)/1.08)-0.001716)/0.997791)
+ 0.166017 * ((1.000000*(($t3 - 2.3976)/1.1988)-0.000474)/0.998470) + 0.424398 *
((1.000000*(($t5 - 2.72)/1.36)-0.000006)/0.999729) + 0.027625 * ((1.000000*(($t64 -
1.0096)/0.5048)-0.000653)/0.999290) + 0.276311 * ((1.000000*(($t73 - 1.592)/0.796)-
0.000242)/0.999613))/1)+0.113931)/1.000000);

print MASS $MASS, "\n";

print "N o r m a l\n";
```

FIGURE 4 *Simplified step 2 inner level perl input file obtained by modifying the DesignFunctions database. The actual input file contains multiple response approximations.*

The probability density functions (PDF) of these variables needs to be parameterized with respect to the outer level variables, as each outer level sample represents a unique variable PDF. Therefore, in addition to the variables automatically detected in the perl script, the nominal design variables and the tolerance scale factor are manually added to the inner level setup as *transfer variables*. These transfer variables added to the inner level *.lsopt* setup file, which is also the outer level LS-OPT stage input file, are automatically detected as parameters in the outer level. These are set as optimization variables in the outer level, but are treated as constant parameters in the inner level. For each outer level sample, the corresponding transfer variable values are substituted in the inner level setup file. The noise variable PDFs in the inner level are parameterized using the transfer variables or their dependents using the "&" operator. The design variables for both inner and outer level LS-OPT analysis are shown in Figure 5 and Figure 6.

For each sample of the outer level, the inner level LS-OPT analysis consists of a Monte Carlo analysis of the Chevrolet C2500 pickup truck to determine the probability of exceeding the crash response bounds. However, it is noteworthy that the expensive transiet dynamic FE crash analysis is replaced by metamodel approximations within the perl input file. This allows significant savings in the computational time. The inner level reliability analysis for probability of failure calculation is set up as a metamodel-based Monte Carlo analysis with 100 space filling sample points (Figure 7).
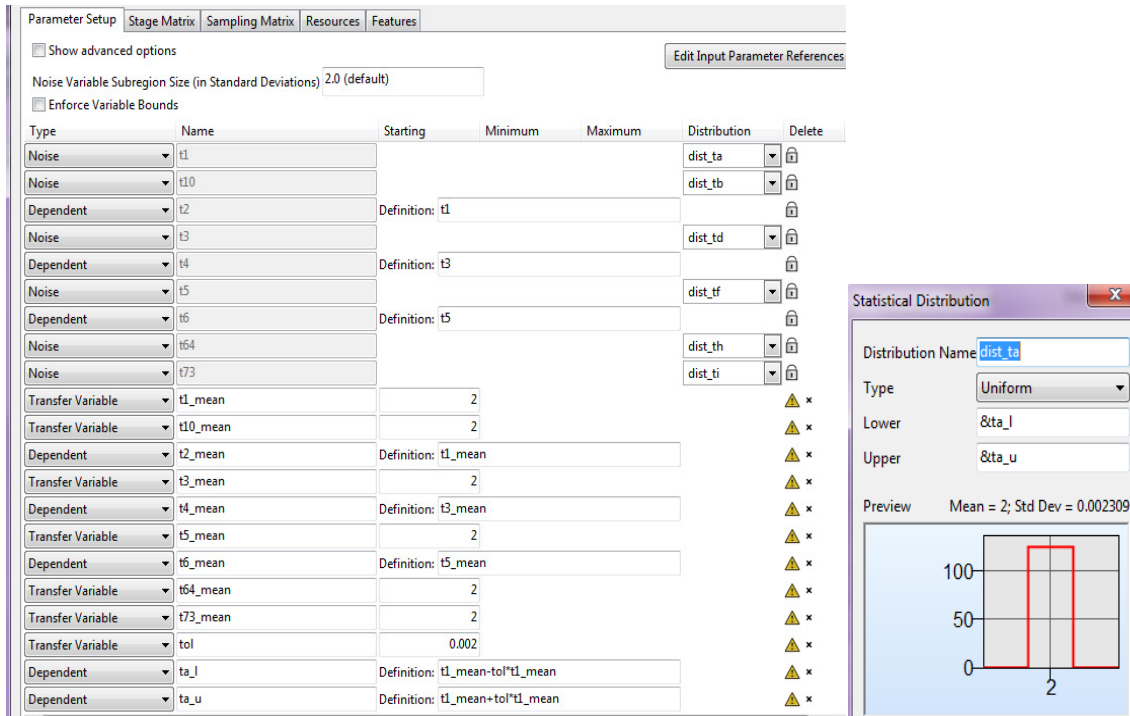
FIGURE 5 Inner level variable setup for step 2 (left). Parameterization of PDF using & operator (right).
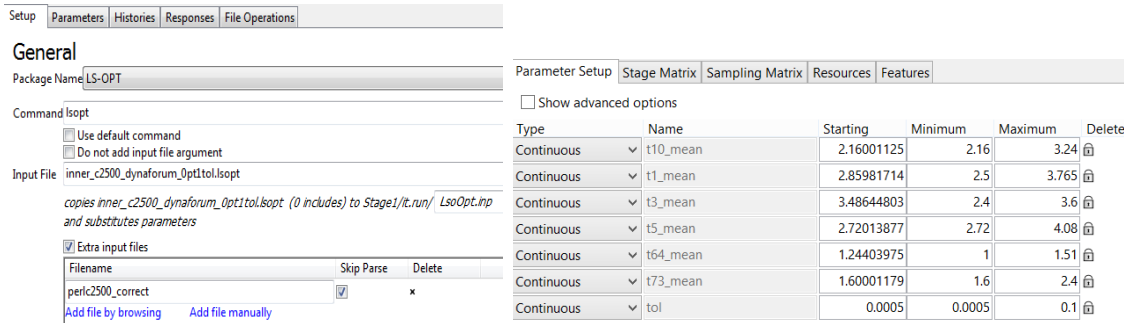


FIGURE 6 Outer level stage setup (left) and variable setup (right)
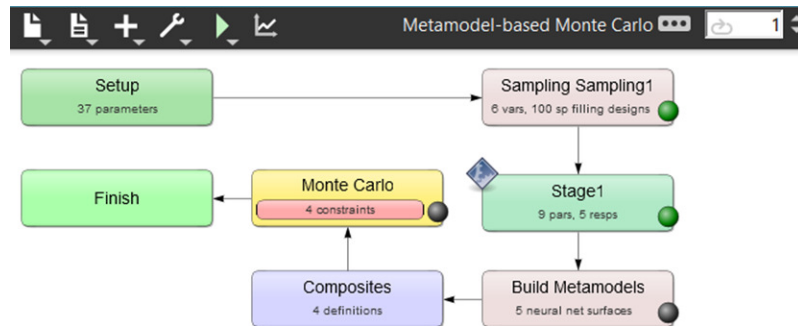


FIGURE 7 LS-OPT setup for the inner loop metamodel-based Monte Carlo analysis.

The four design constraints defined in the Monte Carlo analysis are the three crash responses (intrusion distance and crash pulses) and a constraint for the total mass of the design parts. The crash responses are required to be less than or at least equal to the baseline values. Since the optimum value of the total mass in step 1 was 0.8323, a relaxed value of 0.9 was given as the upper bound for the total mass. It is important to introduce this additional mass constraint in order to ensure a minimum level of mass reduction within the entire tolerance interval. Figure 8 shows the constraint definition of the inner level Monte Carlo analysis.
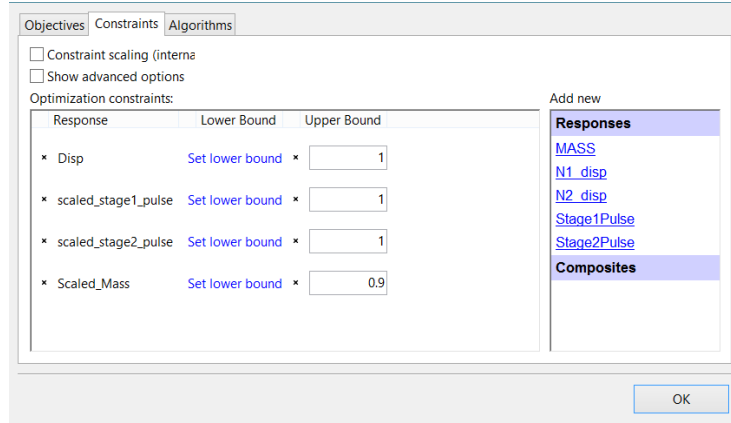


FIGURE 8 Constraint definitions for inner loop Monte Carlo analysis.

The result of the Monte Carlo analysis is a list of probabilities of failure of the crash responses given by the probabilities of exceeding the constraint bounds. Since each inner level Monte Carlo analysis corresponds to one sample of the outer level, the probabilities of failure of the crash responses are defined as the responses of the outer level LS-OPT stage. The values of the probabilities can be obtained from the *lsopt_report* file generated during the inner level Monte Carlo analysis. Since *lsopt_report* is a text file, GenEx response extraction tool of LS-OPT was used to extract the values of probability of failure. Apart from the probabilities of failure, the mean total mass of the parts was also extracted from the inner level. Once the responses of interest were defined for the outer level, a multiobjective optimization to maximize the design tolerance and minimize the mean total mass was formulated as:

$$\max_{t,\mathbf{x}} \quad \{t, -scaled\_mass(\bar{\mathbf{x}})\}$$

$$s.t. \quad P(scaled\_mass(\mathbf{x}) > 0.9) \leq P_{target}$$
$$P(scaled\_stage1\_pulse(\mathbf{x}) > 1) \leq P_{target}$$
$$P(scaled\_stage2\_pulse(\mathbf{x}) > 1) \leq P_{target}$$
$$P(scaled\_disp(\mathbf{x}) > 1) \leq P_{target}$$

$$(5)$$

where $t$ is the tolerance scale factor and $scaled\_mass(\bar{\mathbf{x}})$ is the nominal mass at $\bar{\mathbf{x}}$. The negative sign indicates that the value of the scaled mass is minimized during the optimization process. $P$ is the probability of failure of the crash responses, evaluated using the inner loop Monte Carlo analysis.

A robust solution is desired both with respect to the objective and the constraints, such that any variation within the tolerance limits does not lead to failure. To alleviate the effect of uncertainties, the tolerance scale factor $t$ is maximized. All the thicknesses are assumed to have the same tolerance scale factor or percentage tolerance. $t$ is, therefore, a scalar. The nominal mass is simultaneously minimized, and the problem is solved as a multi-objective optimization to obtain a trade-off between nominal mass and the tolerance. Figure 9 shows the LS-OPT outer level objective function and constraint definitions. The setup of the outer level task, which is set as direct optimization, is shown in Figure 10.
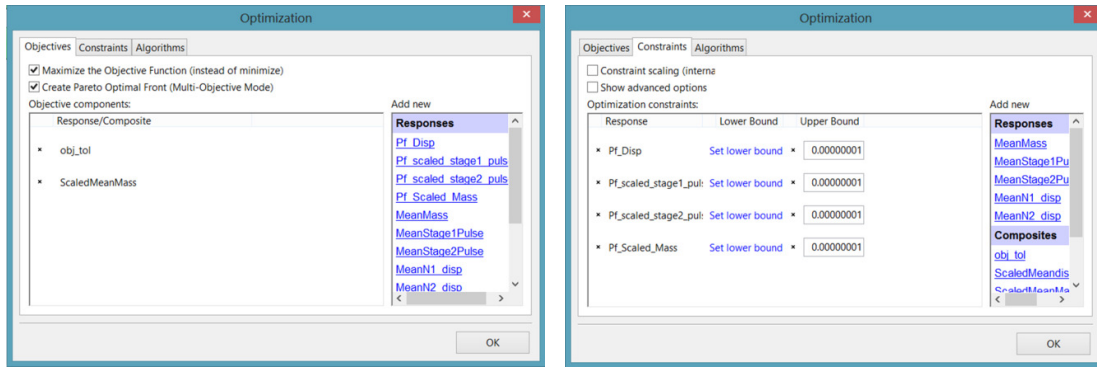
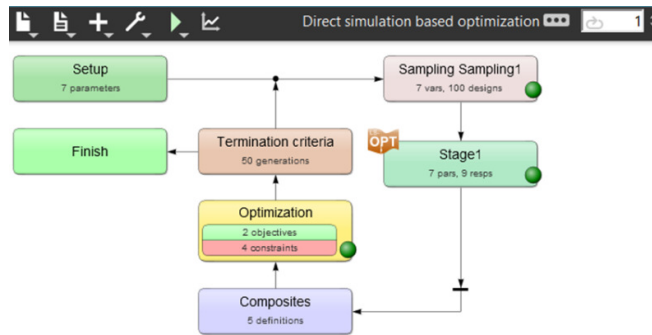FIGURE 9 LS-OPT setup for outer loop objectives (left) and design constraints (right).



FIGURE 10 LS-OPT setup for outer loop design optimization.

## 4 Results and Discussion

The multi-objective optimization problem given in equation 5 and shown in Figure 10 was solved using the *direct simulation based optimization* task of the LS-OPT with genetic algorithm as the optimization technique. A population size of 100 designs per iteration was considered with a maximum of 50 iterations. Thus, 100 outer loop samples were analyzed during each outer level iteration, with each sample consisting of its own inner level Monte Carlo analysis. A Pareto optimal front obtained using the NSGAII method available in LS-OPT was used to determine the tradeoff between the objective functions, i.e. tolerance scale factor vs. the nominal mass as shown in Figure 11 and Figure 13.
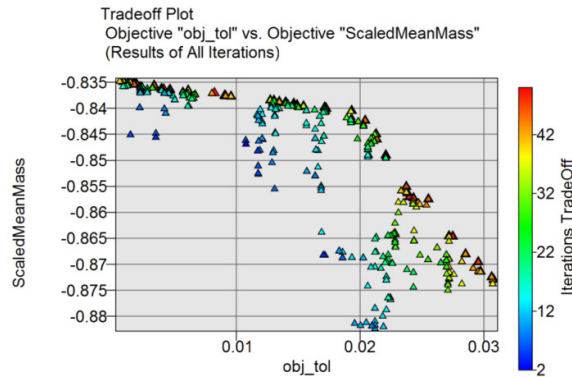


FIGURE 11 Tradeoff between the objective functions over iterations.

The optimization process reached convergence at 50 iterations as shown by the change in hyper volume and spread over iterations in Figure 12. The final Pareto Optimal Front consisting of the nondominated solutions is shown in Figure 13. Since the evaluation of crash responses in the inner loop is based on the approximate analytical equations obtained from step 1, all the non-dominated design points of the Pareto Optimal Front obtained from the multi-objective optimization were analyzed through LS-DYNA FE analysis to determine the true nominal crash responses. Figure 13 shows both the metamodel-based and simulations-based Pareto Optimal Front. The simulation-based Pareto Optimal Front shows that the most Pareto Optimal points of the multi-objective optimization have satisfied the crash response design criteria. A few infeasible design points in the simulation-based Pareto front are due to the error in prediction accuracy of the approximate analytical equations obtained in step 1 that could arise due to some inherent noise in the problem. An interesting thing to note, however, is that the infeasibility of nominal design is observed only for some of those Pareto Optimal points that have low tolerance values. This is an expected result as a higher tolerance mitigates some of the effects of approximation/solution process inaccuracies also. This further underscores the importance of tolerance-based design optimization compared to deterministic optimization.
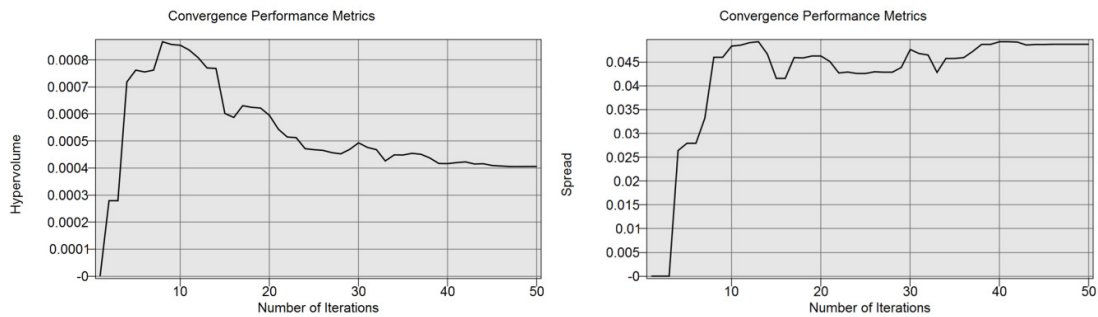


FIGURE 12 Hyper volume change (left) and change in spread (right) of the multi-objective optimization showing convergence.
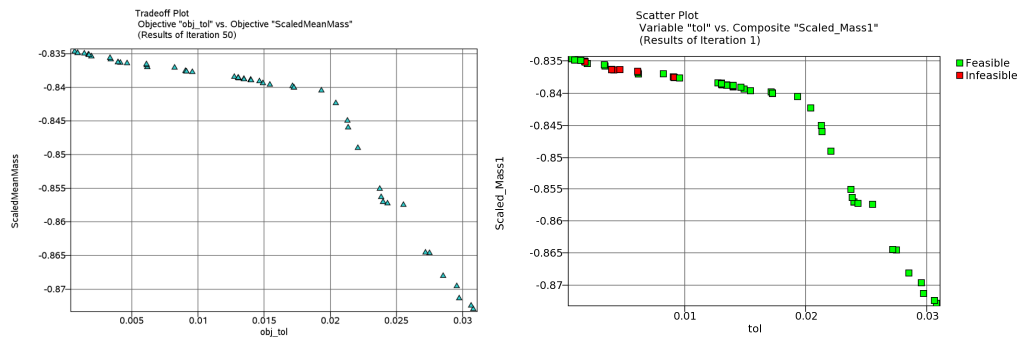


FIGURE 13 Metamodel-based (left) and simulation-based (right) trade-off between tolerance and scaled mass.

The Pareto Optimal Front (Figure 13) shows that a minimum mass solution of approximately 0.835 scaled mass is obtained, 1 being the baseline mass of the selected parts. This corresponds to a vehicle mass reduction of 22.8 kg. Among the Pareto Optimal points obtained, the mass reduction varies between 17.54 kg and 22.8 kg. A maximum tolerance solution of about 0.03 or 3% tolerance is obtained. A knee is observed in the tradeoff plot and it is seen that approximately 2% tolerance can be achieved without significant increase in the mass; the corresponding increase in mass is a mere 0.8 kg.

## 5  Summary and Future Work

A methodology for application of LS-OPT to multilevel tolerance-based optimization is presented. It has been sucessfully applied to optimize the FE model of Chevrolet C2500 pickup truck. A set of Pareto Optimal solutions is obtained with a trade-off between the tolerance and the nominal mass of the vehicle. Significantly higher robustness is achieved as a result of considering tolerances. A maximum tolerance of 3% has been obtained using the optimization technique. At the same time, a mass reduction varying between 17.54 kg and 22.8 kg has also been achieved compared to the baseline. In order to reduce the overall computational cost, the expensive transient dynamic finite element analysis of the truck was replaced by simpler analytical equations obtained using the metamodeling techniques.

The usability of LS-OPT can be improved using two enhancements to the current software. First, the user-defined solver that uses analytical formula previously generated using LS-OPT will be replaced by a metamodel import feature in the near future. In addition, it is also possible to introduce a new task for tolerance optimization that will eliminate the need for a multilevel setup, replacing it with a simpler single level setup.

## 6  References

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