Modified Dynamic Time Warping for Utilizing Partial Curve Data to Calibrate Material Models

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Abstract

Material calibration can be solved as a non-linear regression problem in which a parametric model of the material test is calibrated to its experimental result. In typical material testing, temporal and/or spatial data is produced experimentally and compared to its computational equivalent. At its basic level a single response comparison consists of two curves which are matched to produce a distance between them. The calibration requires minimizing the distance measure. The difficulty of the comparison is determined by phenomena such as noise, hysteresis and differences in geometric curve length (length compatibility). While noise and hysteresis problems have been solved in this context using LS-OPT® in the distant past, the question of curves having substantially different lengths has remained a challenge until recently. In one example, the computational output extracted from LS-DYNA® causes parts of the output to not be relevant to the test data. In this case most distance measures produce spurious distance calculations. This paper introduces a method to address this question. The approach is based on a modification of the Dynamic Time Warping distance measure and referred to as Modified Dynamic Time Warping or DTW-p (for partial). It consists of the trimming of the DTW path as well as iterative mapping to produce a uniform map. An example based on output of the GISSMO model in LS-DYNA is used to demonstrate the effectiveness of the method.

Introduction

Parameter estimation of material models is a crucial component of structural analysis and design. It is used to characterize materials based on constitutive models available in Finite Element analysis software such as LS-DYNA. To accommodate this need, the development of special LS-OPT features for parameter estimation has been ongoing since 1999 when the Euclidean distance norm was firstly introduced as part of the optimization procedure. The many features developed over time were recently extended with Digital Image Correlation (DIC) by making use of full-field spatial data obtained using optical methods. In earlier papers [1,2,3] and the User's Manual [4] some of the available similarity measures as well as the LS-OPT DIC methodology are discussed in significant detail, demonstrated by example.

The existing measures for curve matching available in LS-OPT are the Euclidean norm (also known as Mean Squared Error) [4], Partial Curve Mapping (PCM) [5], Discrete Fréchet (DF) [4] and Dynamic Time Warping (DTW) [6,7]. PCM was introduced in 2011 to address hysteresis and partial curves (in which the lengths are incompatible) with some success, except that noisy curves rendered the method ineffective. Noise occurs in computational results mainly due to the discrete nature of element erosion algorithms. This prompted the implementation of DF and DTW, which were designed to compare noisy curves end to end. They did so successfully if the curves being compared were of comparable geometric length. The various phenomena which may occur when comparing test curves are illustrated in Figures 1 a-c. The crosses represent the test curve in each case.

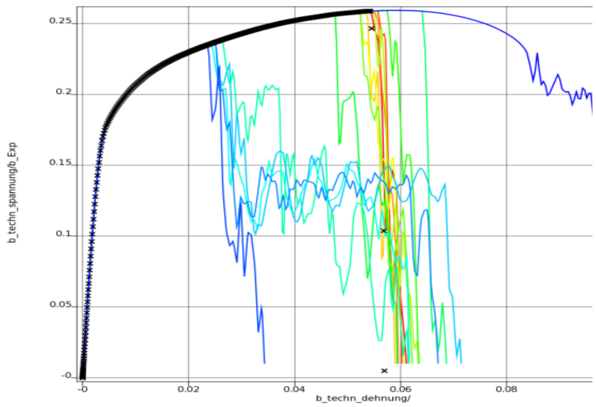


Fig. 1a: *Noise:* Force-displacement response of the GISSMO failure model (LS-DYNA). The differently colored curves represent different material parameter sets.

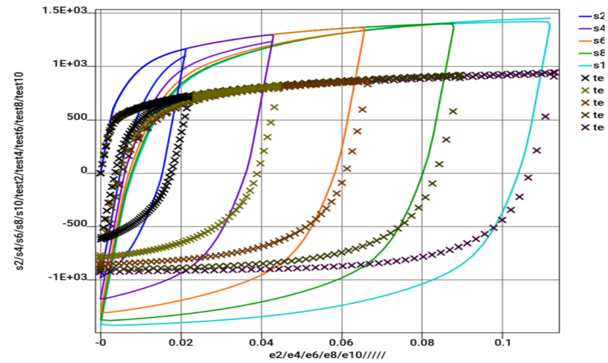


Fig. 1b: *Hysteresis:* Force-displacement response of LS-DYNA Material 125 loading and unloading. The differently colored curves represent different load cases.

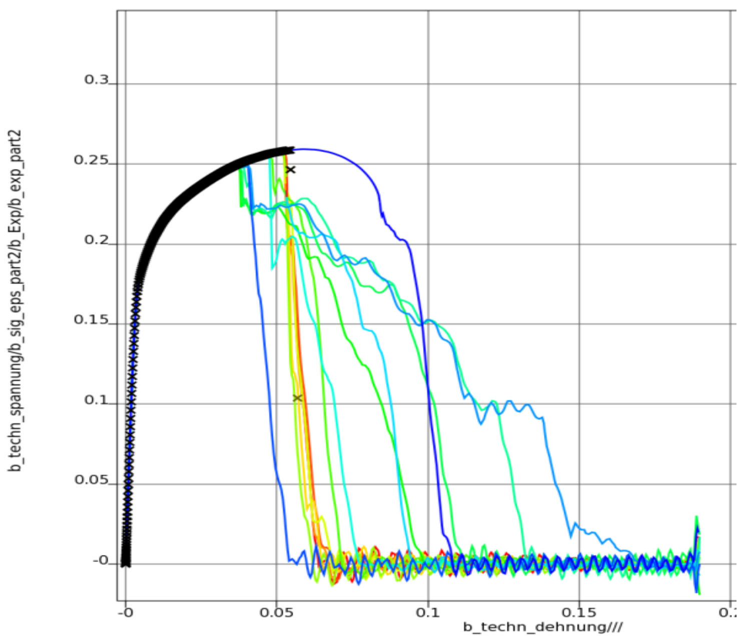


Fig. 1c: *Curve incompatibility* caused by post-failure oscillation of the coupon. The crosses represent the test curve with rapid failure at about 0.05. The differently colored curves represent different material parameter sets. The curve segment oscillating about the zero line simulates vibration of the fractured coupon.

Fig. 1: Phenomena which typically challenge the robustness and accuracy of similarity measures for material calibration. The black crosses represent the experimental curve. Figures produced by LS-OPT.

Table 1 summarizes and compares all the methods available in LS-OPT 6.0. None of the methods were able to robustly address examples displaying a combination of noise, hysteresis, and length incompatibility. Reference [9] can also be consulted for research on the finer detail of comparing the available methods. In this reference, Jekel also introduces a new method, the Area method, although it is suspected it might also suffer from the length compatibility defect associated with DTW since it is an end-to-end method which computes the area between the curves.

Table 1: Comparison of the similarity measures available in LS-OPT 6.0

Distance Metric	*Principle*	*Advantage*	*Disadvantage*
Euclidean Distance (MSE)	Mean Squared Error of the difference in curve ordinates	Handles noise, partial curves	Cannot deal with hysteresis: the curves must be functions such as time histories.
Partial Curve Mapping (PCM)	Area between curves. Preserves the arc length.	Handles curves of unequal geometric length and hysteresis	Cannot deal with noise. Due to arc length preservation.
Discrete Fréchet (DF)	Intuitive definition of a man and dog traversing the two curves. The distance is the length of the shortest leash. Speed can vary, but neither can move backwards.	Handles noise and hysteresis	Cannot deal with curves of unequal geometric length (incompatible).
Dynamic Time Warping (DTW)	Minimizes the sum of *path* connectors between the curves in a one-to-many mapping end to end.	Handles noise and hysteresis	Cannot deal with incompatible curves. More complicated to code. More expensive to compute.

In practical situations, curves are typically not of comparable geometric length. While the test curve available from experimental data can be edited to any desirable length, the computational curves produced in the optimization cycle cannot, or can only be edited with difficulty. In DIC-based problems, there could be hundreds of thousands of curves which may not be easy to handle with a single criterion. Examples of length incompatibility are:

1. A tensile test in which the coupon fractures but then the analysis continues to a fixed termination time with the only response being the oscillation of the two remaining fractured pieces. Computational runs are often scheduled conservatively to extend much beyond fracture and total separation of the fractured parts.
2. An example in which only a part, e.g. post-yield, of a curve needs to be matched.

While the PCM method is available for addressing curve partiality, the method has trouble with noisy curves because of its inherent property of preserving the arc length as a means of traversing the curve.

A search for similarity measures able to address curve incompatibility, in combination with other properties such as noise and hysteresis, was recently incorporated as part of a master's thesis study [10] on optimization at the Technical University in Darmstadt, Germany. The study found the Time-Warped Longest Common Subsequence (T-WLCS) method [8,11] to be a promising approach to partial curve comparison. Based on T-WLCS, the points of simulation and test curves are traversed and labeled as a match whenever the points are within ε-range of each other. In doing so, a percentage of mismatch between the curves is calculated. The curve(s) with the lowest percentage of mismatch with respect to the experimental curve are determined and, if necessary, appropriately truncated. Since the percentage of mismatch is dependent on the matching threshold ε, more than one curve may have the lowest percentage of mismatch. To address this issue, a hybrid method was proposed [12] in which the second step consists of passing these (potentially truncated) matches to the DTW measure in order to determine the (partial) computed curve that best matches the experimental data. The proposed hybrid measure does not require aligned x-values and allows matching one point from a curve to multiple points on the other. In contrast to measures such as DF and DTW, the proposed hybrid measure does not require all points on both curves to be matched, which yields an automated truncation, whenever there is noise or different curve lengths [12].

Despite the initial promise, the study concluded that a pitfall of the T-WLCS minimization method was the difficulty of choosing a suitable ε value (geometric tolerance). A large value is bound to allow spurious data, such as the oscillating tail, to be partially ignored. On the other hand, choosing an ε value which is too small may distort the actual curve due to gaps, thereby artificially inflating the DTW distance value. Hence this method was found to be unsuitable for practical use in an industrial environment.

In the current study, the original Dynamic Time Warping method of Berndt and Clifford [6] was revisited and, rather than utilizing the LCS principle, a simple modification of the DTW method was implemented with the purpose to accommodate curve incompatibility. In the modified method, DTW-p, all, but one, of the path end connectors sharing a common test point are trimmed to yield a partial computed curve. This DTW modification is significantly simpler than the hybrid T-WLCS approach and requires no user parameters.

A tensile test is used to demonstrate the new method. The simulation data for this example is obtained by means of the finite element software LS-DYNA using the GISSMO damage model. Experimental data from a simulated tensile test are used to identify seven material parameters of the model. The example exhibits both noise and mismatch of the curve length due to an uncertain termination point caused by oscillation after fracture.

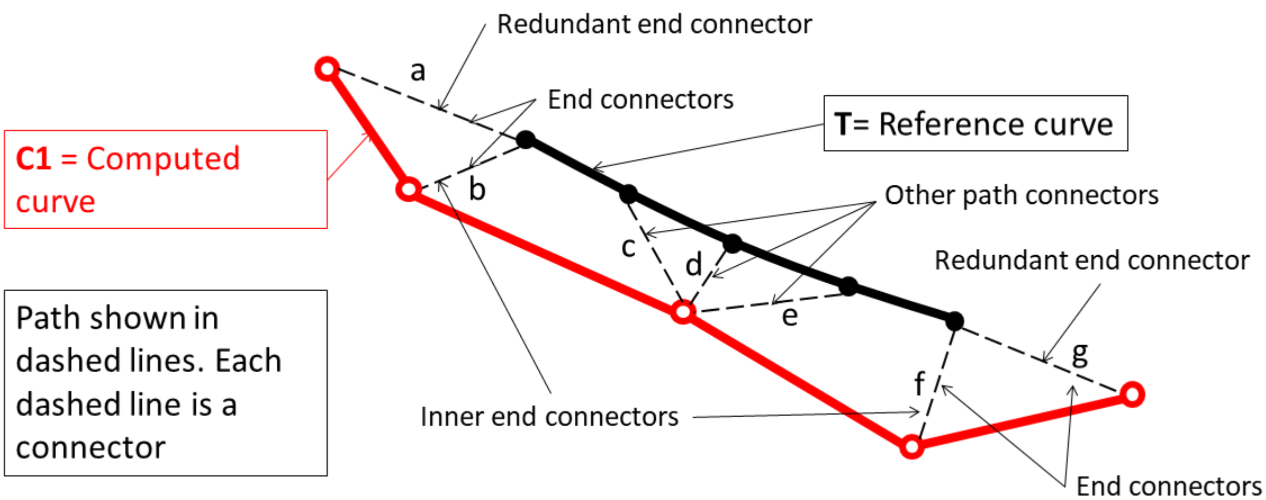
Methodology

The goal of the research was to find a compatible matching curve pair by trimming the original computed curve and then to compute the distance between the reference curve and the remaining *compatible* computed curve segment using Dynamic Time Warping. The algorithm is as follows:

1. Assume a reference curve and a computed curve. Typically, multiple computed curves are compared to a fixed curve (the experimental curve). This may be part of an optimization process to find the computed curve closest to the experimental curve.
2. The DTW method is used for distance calculation. As part of this calculation, it produces a "path" which is a set of connectors between its vertex pairs. The mapping can be one-to-many.
3. The DTW distance is computed by adding up the connector lengths and dividing by its size (number of connectors).

4. DTW measures curves *end to end*, so if one curve is much longer than the other although part of it is a perfect match, the DTW distance can be large. In this case there are typically (though not necessarily) multiple end connectors connected to at least one end (here designated as *fanning*). An end connector emanates from the end point of the experimental curve. *Definition:* Mathematical incompatibility of curves occurs when the DTW path has redundant end connectors. This signifies that the one curve is "longer" than the other (incompatible).
5. Some end connectors are redundant. These are connectors emanating from the end points of the experimental curve, except the innermost one at each end, which is kept for connecting the end points of the experimental and computed curves.
6. The goal is to recursively trim curves with redundant end connectors until there remain no redundant end connectors. Then the curves are defined to be compatible and the distance between them can be computed using the standard DTW measure.
7. To make the algorithm function smoothly, the computed curve is remeshed in each cycle to have the same length (in terms of the number of nodes) as the reference curve. This helps to regularize the comparison as each curve has the same number of points. Bisectional interpolation can be used for remeshing to preserve the original curve properties.
8. The algorithm is terminated once there are no redundant end connectors. Then the curves are defined to be compatible. The computed DTW distance is then the final distance between the compatible curves.

The method is illustrated as follows:



1. Compute DTW (Dynamic Time Warping) path between T and C1. Distance between T and C1 = sum of path connector lengths = *(a+b+c+d+e+f+g)/n*

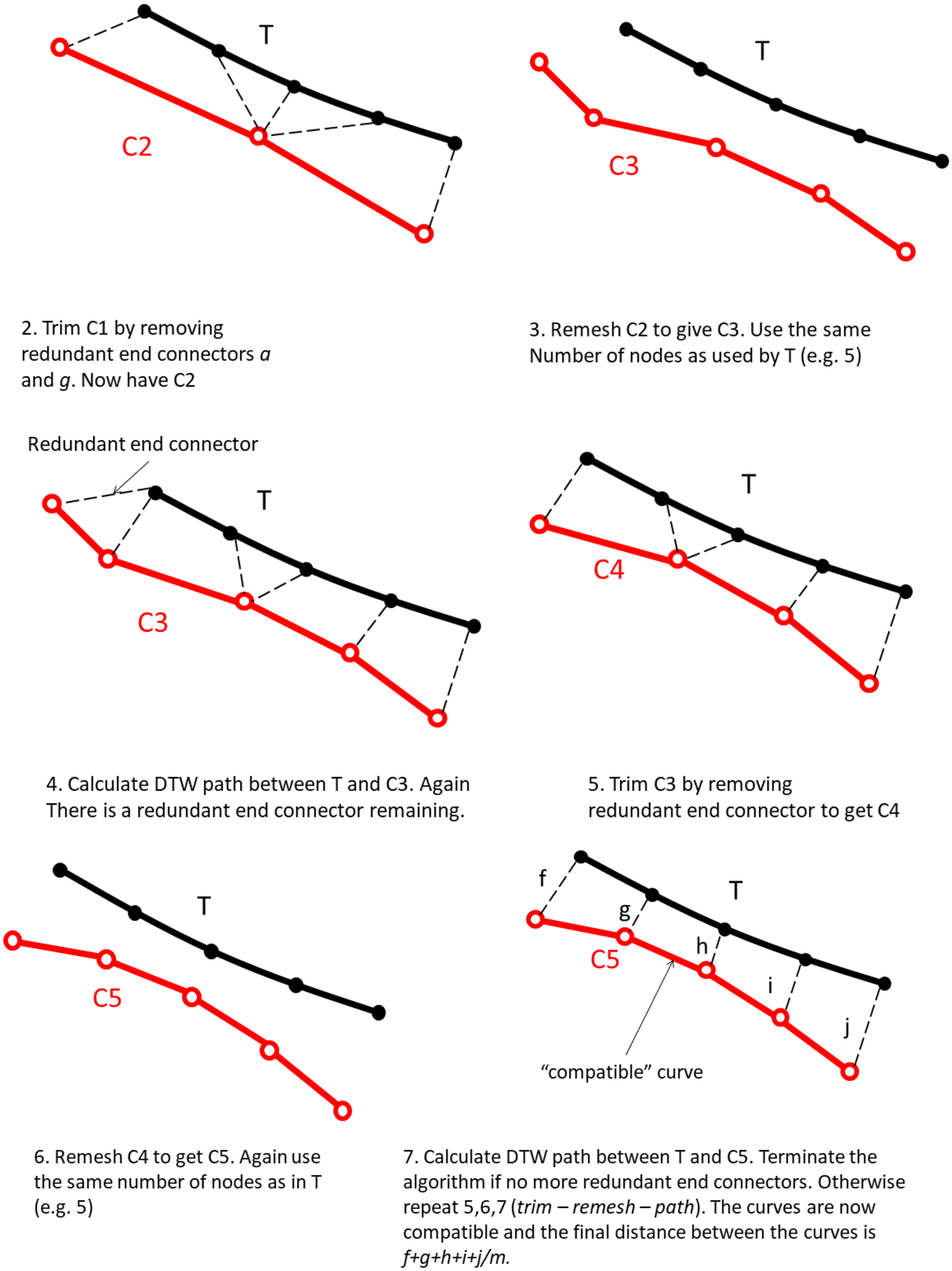


Fig. 2: Schematic of the truncation steps

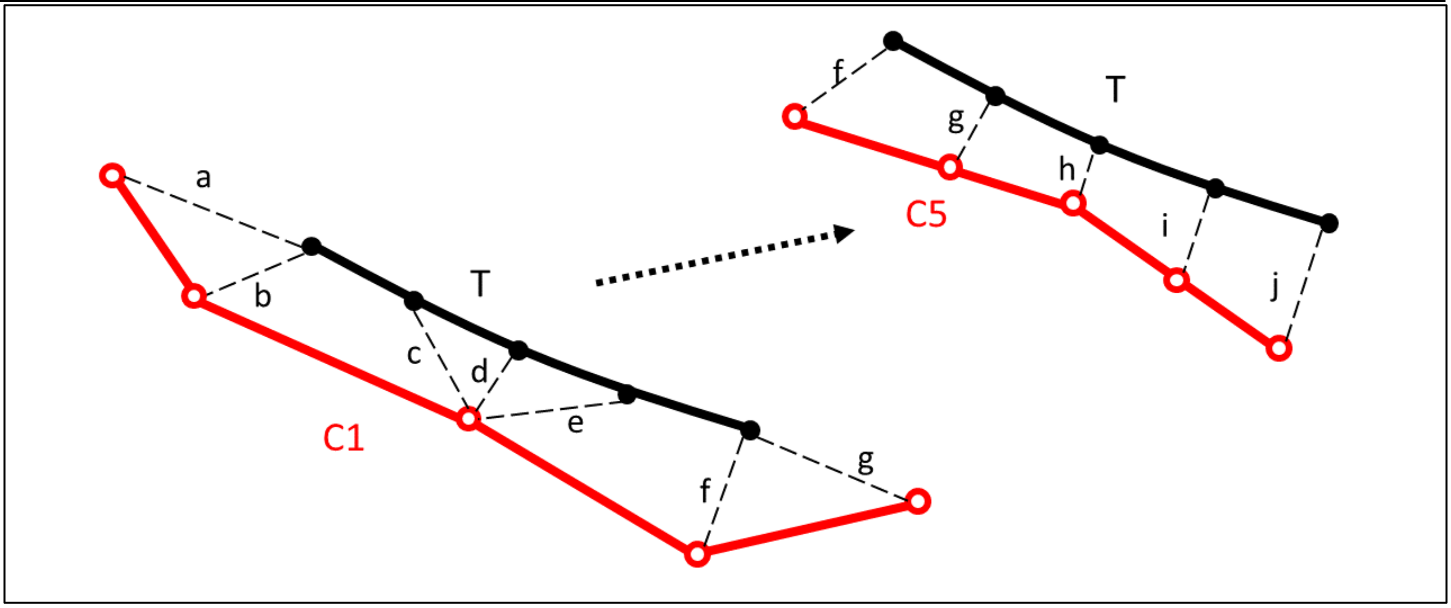


Fig. 3: Summary of the evolution of the computed curve.

LS-OPT Implementation

Table 2 shows the three functions implemented. The curves are normalized to the reference curve before applying the mapping. The resolution of the DTW-p mapping can also be increased by specifying the number of points required. The methodology was implemented in a development version destined as LS-OPT Version 6.1. The functions and attributes can of course be selected from the LS-OPT GUI.

The truncation loop is part of the parallel extraction process in LS-OPT.

Table 2: Truncated Dynamic Time Warping functions implemented in LS-OPT.

Function name	Function type	Description
`MultiHistDTWP`	Scalar	Computes the *DTW-p* distance for single and spatial histories. This function is typically minimized during calibration. This function is also used for calibration using *Digital Image Correlation*.
`DTWPHistory`	Vector	Computes the truncated curve using the *DTW-p* function. This data is used to display the truncated curve.
`DTWPMultiHistory`	Array (vectors at multiple points)	Computes the truncated spatial history using the *DTW-p* function.

Example and observations

An optimization was conducted on a tensile test using the GISSMO material model available in LS-DYNA. Fig. 4 shows the full optimization history with the optimum curve in magenta.

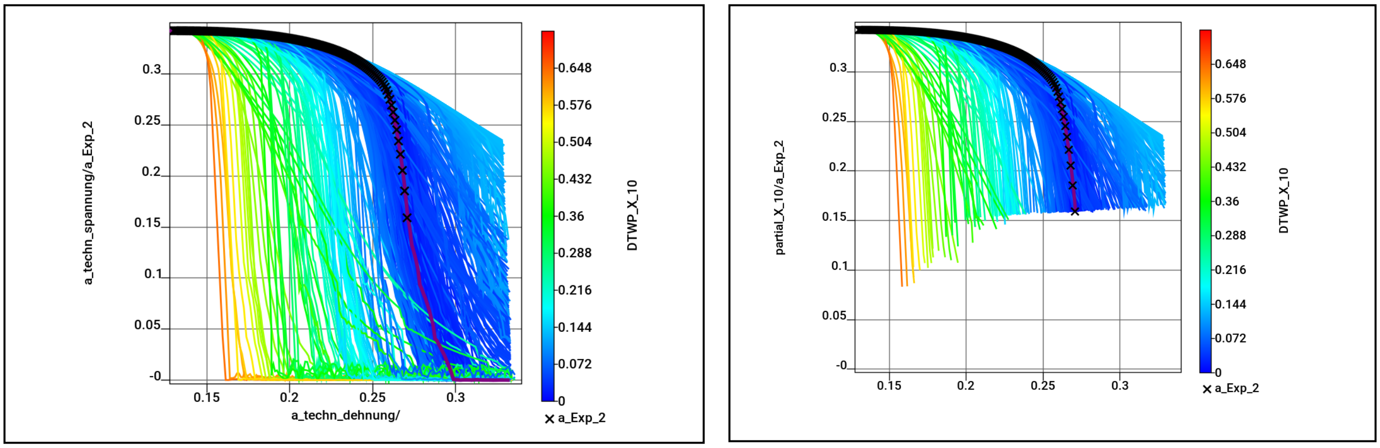


Fig. 4: Full (left) and truncated curve sets (right) for LS-DYNA GISSMO model (7 parameters). Each individual curve represents a candidate point in the material parameter space. The figures represent the same parameter set. The curves are color coded to represent the DTW distance (in the range [0,0.7]). The heavy magenta line represents the optimum curve closest to the experimental results (black crosses). The truncation algorithm to produce the curves on the right typically represent fewer than 10 internal truncation loops. (Figures produced by LS-OPT.)

Conclusion

A modified Dynamic Time Warping algorithm was implemented in LS-OPT to truncate computed curves with lengths which are incompatible with the reference (experimental) curve. The target problem is the case in which the best match to an experimental curve cannot produce a small DTW distance value mainly due to incompatible end point(s). It is assumed that noise and/or hysteresis might also be present. An example consisting of a large curve set shows that the method effectively truncates incompatible curves.

Other conclusions are:

- DTW-p is better than the formerly presented T-WLCS [11] because it does not have any user parameters that need to be tweaked.
- An additional benefit is that it renders partial curves of similar length. This causes the distance values to vary rationally, not influenced by the value of the parameter ε. I.e. DTW-p should produce a smoother function of the parameters than T-WLCS.
- The modification of the DTW formulation is simple and the recursive method used to ensure curve compatibility is convergent and typically takes fewer than 10 iterations of an internal parallel loop.

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