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ON THE ROBUSTNESS OF A SIMPLE DOMAIN REDUCTION SCHEME FOR SIMULATION-BASED OPTIMIZATION

Nielen Stander[1] and K.J. Craig[2,*]
[1] Livermore Software Technology Corporation,
7374 Las Positas Rd, Livermore, CA 94550, USA
nielen@lstc.com
[2] Multidisciplinary Design Optimization Group (MDOG), Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria 0002, South Africa
ken.craig@eng.up.ac.za

* Professor. Currently on sabbatical leave at Livermore Software Technology Corporation.

Abstract

This paper evaluates a Successive Response Surface Method (SRSM) specifically developed for simulation-based design optimization, e.g. that of explicit nonlinear dynamics in crashworthiness design. Linear response surfaces are constructed in a subregion of the design space using a design of experiments approach with a *D*-optimal experimental design. To converge to an optimum, a domain reduction scheme is utilized. The scheme requires only one user-defined parameter, namely the size of the initial subregion. During optimization, the size of this region is adapted using a move reversal criterion to counter oscillation and a move distance criterion to gauge accuracy. To test its robustness, the results using the method are compared to SQP results of a selection of the well-known Hock and Schittkowski problems. Although convergence to a small tolerance is slow when compared to SQP, the SRSM method does remarkably well for these sometimes pathological analytical problems. The second test concerns three engineering problems sampled from the nonlinear structural dynamics field to investigate the method's handling of numerical noise and non-linearity. It is shown that, despite its simplicity, the SRSM method converges stably and is relatively insensitive to its only user-required input parameter.

Keywords: Simulation-based optimization, response surface methodology, multipoint approximations, design of experiments, crashworthiness.

Introduction

The success of finite element simulation to augment or even replace physical experimentation in design has accelerated the development of simulation-based optimization in recent years. While having its origins in the statistics of physical experimentation, response surface methodology (RSM) (Box & Wilson, 1951, Myers and Montgomery, 1995) has been the primary gradient-free simulation-based approach available. The general unavailability of analytical gradient information in analysis codes arises from the complexity of the non-linear finite element formulation. While not requiring any code enhancement, an alternative approach by means of finite differences may result in spurious gradients, not suitable for gradient-based optimization. For these reasons, and because of the noise-filtering properties of RSM, it has become particularly popular for impact design applications such as crashworthiness or metal forming where the response can be highly nonlinear.

As analysis methods for impact dynamics began to take hold in industry in the late eighties, design optimization methods of impact design followed in the mid 1990's. Among the topics studied are occupant safety (Etman *et al*, 1996, Etman, 1997), component-level optimization (Marklund, 1999, Akkerman *et al*, 2000), airbag-related parameter identification (Stander, 2000) and full-vehicle simulation (Sobieszczanski-Sobieski et al, 2000). The response surface method appeared in several forms, e.g. a successive response surface method (Toropov, 1989, Etman *et al*, 1996, Kok & Stander, 1999, Stander, 2001) and an updated response surface method (Schramm & Thomas, 1998, Sobieszczanski-Sobieski *et al*, 2000). Toropov (1989) experimented with linear and multiplicative approximations for his iterative multipoint approximation method and applied weighted least squares fitting and reduction of the subregion size based on function accuracy. In later work, Toropov presented refinements of his method in the form of indicators for move limit strategies. These criteria have been incorporated in a multipoint approximation strategy known as MARS (Toropov, 1998). The methodology of Etman (1997) uses a successive linear approximation approach with a saturated experimental design ($n + 1$ points, with n the number of design variables) within a subregion of the design space. To determine the location and size of each new subregion, a complex heuristic is used, based on oscillation, the accuracy of the response surface and constraint activity. More recently, Sobieszczanski-Sobieski et al (2000) conducted a full-vehicle simulation of a multidisciplinary nature while using a single set of higher-order response surfaces. In a metal-forming application Kok & Stander (1999) used a successive linear response surface method while Akkerman *et al* (2000) demonstrated the use of a similar but slightly enhanced successive approximation method to a knee bolster design with shape variables and involving transient mesh adaptivity.

While these studies demonstrate optimization capability by means of examples, there appears to be a dearth of studies that assess accuracy and robustness in design optimization in nonlinear dynamics. Against this background, the present paper outlines a simple, dual criterion successive response surface method (SRSM) that requires a single user-defined parameter. Furthermore, a deeper investigation is conducted into the convergence properties of the method (SRSM) as applied to a large set of algebraic test problems as well as a smaller set of simulation-based problems. For the algebraic problems, the SRSM method is compared to the more standard Successive Linear Programming (SLP) method where both use the same adaptive domain reduction approach.

The motivation for the method proposed in the paper is derived from the requirements for simulation-based optimization (Craig & Stander, 2001):

1. *Robustness and accuracy*. In practical applications, it is important that the optimization method produces an answer to engineering accuracy or at least an immediate and significant improvement of the objective.
2. *Efficiency*. The number of expensive simulation-based function evaluations required for each design iteration must be limited. Direct optimization methods without approximations or evolutionary algorithms like the genetic algorithm are usually disqualified due to the large number of function evaluations required.
3. *Parallelization*. To improve efficiency, modern simulations run on multiple computers and/or processors. The optimization method must therefore be parallelizable. This disqualifies e.g. sequential line searches.
4. *Noise*. The step-size dilemma of gradient-based methods must be addressed as this impacts both robustness and efficiency. A noise filtering capability may avoid local optima.
5. *Infeasibility*. The algorithm must be able to start from and handle intermediate infeasible designs if they can be simulated. It must also be able to provide a best compromised design if no feasible design is possible within the constraints specified.
6. *Global optimum.* This requirement is probably the strictest of all those listed. If an algorithm has features that at least provide the possibility of not terminating on the first local optimum it finds, then this will be desirable in practical applications. The study of true global optimization algorithms lies outside the scope of this paper.
7. *Ease of use.* The number of user-selected parameters must be kept to a minimum.

A method that successfully addresses most of these requirements is the Successive Response Surface Method (SRSM) based on *oscillation* and *move distance* criteria and first described in Stander (2001). This algorithm uses RSM (Myers & Montgomery, 1995), i.e. a Design of Experiments approach, to construct linear response surfaces on a subregion from a *D*-optimal subset of experiments. Linear functions are used to minimize the number of simulations required, especially for a very large number of variables. Successive subproblems are solved using a multi-start variant of the dynamic trajectory method, LFOPC (Snyman, 2000). To select the optimum, multi-starts are performed from the locations coinciding with the subset of experimental design points. The size of each successive subregion is adapted based on contraction and panning parameters designed to alleviate oscillation and prevent premature convergence. To prevent remote designs from affecting the accuracy of the subregional optimum, simulation results from previous iterations are not incorporated and each response surface is strictly based on the results of a *D*-optimal experimental design within the current subregion. Infeasibility is handled automatically when it occurs through the construction and solution of an auxiliary problem to bring the design within the subregion if possible. The method handles noisy responses automatically through the selection of an initially large subregion and a typically 50% over-sampling of experiments in the implementation of the *D*-optimality criterion (Roux, Stander & Haftka, 1998). As the optimum is approached, the subregion is contracted automatically, implying that inaccuracies in the sensitivity information do not cause large departures from the previous design. Therefore this handling of the *step-size dilemma* (Haftka & Gürdal, 1990) also provides an inherent move limit to the algorithm. The use of an adaptive subregion or trust region is not new, e.g., in Lin *et al* (2000), Pérez *et al* (2000), and Alexandrov *et al* (1997), the ratio of the simulated (actual) objective function reduction to that of the approximated objective function reduction in each design step is used as a measure to adjust the trust region size.

The SRSM method has proved itself to be robust but only moderately efficient if convergence to a tight tolerance is required. The over-sampling required for each response surface, although fully parallelizable, implies that it requires 50% more function evaluations for each design iteration than the minimum required by gradient-based algorithms. This method, although by no means a global optimization algorithm, may be more likely to find a lower local optimum than local approximation (gradient) methods due to its 'wider' perspective of the design space as embodied in the response surface. However, experimentation with multi-start designs on suitable test problems is required to verify this.

The aim of this study is also to illustrate that the SRSM method provides an accurate yet efficient and robust optimization methodology to address both smooth and noisy simulation-based problems. The test cases are therefore chosen accordingly and are grouped in two main categories. The first is a random collection of analytical and sometimes pathological problems from Hock & Schittkowski (1981) that are often used for testing optimization algorithms. These examples possess reliable gradient information, so one would expect a good local approximation method to perform well. The second category contains simple but general structural optimization problems for testing the algorithm's ability to handle practical engineering problems. These are a nonlinear explicit dynamic crash optimization problem of a simplified car, a material identification problem that employs the nonlinear implicit analysis of a tensile test specimen, and an occupant safety-related head impact problem. The problems in the second category exhibit various degrees of noise and nonlinearity and are therefore ideal to demonstrate the handling of these characteristics.

Methodology of Successive Response Surface Method (SRSM)

Consider the general nonlinear optimization problem:

$$\text{Minimize } f(\boldsymbol{x}), \ \boldsymbol{x} \in R^n \tag{1}$$

subject to the inequality constraints

$$L_j \le g_j(\boldsymbol{x}) \le U_j; \quad j = 1,2,\ldots,m \tag{2}$$

and simple bounds on the design variables

$$x_{il} \le x_i \le x_{iu}; \quad i = 1,\ldots,n \tag{3}$$

where L_j and U_j refer to the upper and lower bounds on each of the inequality constraints, and x_{il} and x_{iu} the lower and upper bounds on each of the design variables, n is the number of design variables, and m the number of inequality constraints. Note that equality constraints can be written as two inequality constraints in the form of Equation 2 with L_j equal to U_j.

Refer to Roux, Stander & Haftka (1998) and Stander (2001) for a detail description of the Successive Response Surface Method (SRSM). The method, as implemented in LS-OPT (Stander, 1999), has a number of features that makes it robust and suitable for the solution of practical problems:

- The *D*-optimal experimental design is used to best utilize the number of available runs. Over-sampling of 50% is used to maximize the predictive capability (Roux, Stander & Haftka, 1998) of the response surfaces.
- Linear approximations are constructed using linear regression on all the points of the current iteration. Unit weighting is used for the regression.
- An adaptive domain reduction method is applied as described in detail below.
- An auxiliary problem that minimizes the maximum constraint violation is solved to enforce feasible designs.

The SRSM method uses a region of interest, a subspace of the design space, to determine an approximate optimum. A range is chosen for each variable to determine its initial size. A new region of interest centers on each successive optimum. Progress is made by moving the center of the region of interest as well as reducing its size. Figure 1 shows the possible adaptation of the subregion.

The starting point $\boldsymbol{x}^{(0)}$ will form the center point of the first region of interest. The lower and upper bounds $(x_i^{rL,0}, x_i^{rR,0})$ of the initial subregion are calculated using the specified initial range value $r_i^{(0)}$ so that

$$x_i^{rL,0} = x_i^{(0)} - 0.5 r_i^{(0)} \text{ and } x_i^{rU,0} = x_i^{(0)} + 0.5 r_i^{(0)} \qquad i = 1, \ldots n \tag{4}$$

where n is the number of design variables. The modification of the ranges on the variables for the next iteration depends on the oscillatory nature of the solution and the accuracy of the current optimum.

A contraction parameter γ is firstly determined based on whether the current and previous designs $\boldsymbol{x}^{(k)}$ and $\boldsymbol{x}^{(k-1)}$ are on the opposite or the same side of the region of interest. Thus an oscillation indicator c may be determined in iteration k as

$$c_i^{(k)} = d_i^{(k)} d_i^{(k-1)} \tag{5}$$

where

$$d_i^{(k)} = 2\Delta x_i^{(k)} / r_i^{(k)}; \quad \Delta x_i^{(k)} = x_i^{(k)} - x_i^{(k-1)}; \quad d_i^{(k)} \in [-1;1] \tag{6}$$

The oscillation indicator (purposely omitting indices *i* and *k*) is normalized as $\hat{c}$ where

$$\hat{c} = \sqrt{|c|}\, sign(c)\,. \tag{7}$$

The contraction parameter γ is then calculated as

$$\gamma = \frac{\gamma_{\text{pan}}(1+\hat{c}) + \gamma_{\text{osc}}(1-\hat{c})}{2}\,. \tag{8}$$

The parameter γ_{osc} is typically 0.5-0.7 representing shrinkage to dampen oscillation, whereas γ_{pan} represents the pure panning case and therefore unity is typically chosen.

The accuracy is estimated using the proximity of the predicted optimum of the current iteration to the starting (previous) design. The smaller the distance between the starting and optimum designs, the more rapidly the region of interest will diminish in size. If the solution is on the bound of the region of interest, the optimal point is estimated to be beyond the region. Therefore a new subregion, which is centered on the current point, does not change its size. This is called panning (Figure 1(a)). If the optimum point coincides with the previous one, the subregion is stationary, but reduces its size (zooming) (Figure 1(b)). Both panning and zooming may occur if there is partial movement (Figure 1(c)). The range $r_i^{(k+1)}$ for the new subregion in the (*k* + 1)-th iteration is then determined by:

$$r_i^{(k+1)} = \lambda_i r_i^{(k)}; \quad i = 1,\ldots,n; \quad k = 0,\ldots,niter \tag{9}$$

where λ_i represents the contraction rate for each design variable. To determine λ_i, $d_i^{(k)}$ is incorporated by scaling according to a zoom parameter η, typically 0.5, that represents pure zooming and the contraction parameter γ to yield the contraction rate

$$\lambda_i = \eta + \left|d_i^{(k)}\right|(\gamma - \eta) \tag{10}$$

for each variable independently (see Figure 2). This criterion replaces function error and feasibility-based criteria frequently employed in earlier response surface formulations (Etman, 1997, Toropov, 1998).

For the Successive Linear Programming (SLP) method used for comparison in the results section, linear response surfaces are constructed using the gradient at the current point. The subregion is centered on this point while its adaptive properties are governed by the same heuristics as the SRSM method.

Test cases

The move limit heuristics of the SRSM and SLP methods are set to $\gamma_{pan} = 1.0$, $\gamma_{osc} = 0.6$ and $\eta = 0.6$ for all the test cases below unless indicated otherwise.

Hock and Schittkowski problems

37 arbitrarily selected Hock problems and one problem from Svanberg (1995, 1999) are used in this benchmark with the same starting designs being used for testing all the algorithms. The problems are all analytical expressions with analytical gradients but the gradients are computed numerically to emulate a simulation-based environment to align the test with the thrust of this paper. Five of the problems (Nos. 2, 15, 16, 17, 20) are variations of the Rosenbrock problem ($f = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$), while the number of design variables ranges between 2 and 21. All the selected problems are constrained optimization problems.

Small car crash problem

This problem (Figure 3) consists of a simplified vehicle moving at a constant velocity of $15.64 m.s^{-1}$ (35mph) and impacting a rigid pole. The nonlinear finite element structural solver LS-DYNA (LSTC, 2000) is used to perform a simulation of the crash using the explicit dynamic analysis method. The simulation duration is 50ms. The objective is to minimize the Head Injury Criterion (HIC) (NHTSA, 2000) over a 15ms interval of a selected point subject to an intrusion constraint of 550mm of the pole into the vehicle at 50ms. This criterion is based on linear head acceleration and was designed to minimize skull fracture/brain injury due to head contacts with the vehicle interior (NHTSA, 2000). The design variables are the shell thickness of the car front (t_{hood}) and the shell thickness of the bumper (t_{bumper}).

Material identification problem (Müller, 2000)

In a material identification problem the optimization process uses experimentally measured data to calibrate a constitutive model. A non-linear simulation is performed with the model parameters as input, and the discrepancy of the simulated and measured results is used as a minimization criterion. In this example, the parameters of a power-law material model of a tensile test specimen are determined using the experimental reaction force, *F* and elongation, *u*. The stress-strain history of the specimen (Figure 4) is simulated using LS-DYNA (LSTC, 2000) and the objective is defined as the least-squares difference between the simulated and measured force-elongation history. The design variables in this problem are the two material parameters in the power-law model, as defined in Equation 11.

$$\sigma_y = K\varepsilon^r = K(\varepsilon_{yp} + \varepsilon^p)^r \tag{11}$$

where ε_{yp} is the elastic strain to yield and ε^p is the effective plastic strain (logarithmic). The strength coefficient, *K* and strain-hardening exponent, *r* are used as design variables.

Head impact problem (Balasubramanyam, 2001)

This problem is outlined in Figure 5. Shown is a Free Motion Headform (FMH) impacting the A-pillar of a vehicle covered on the interior with plastic trim. The aim of the optimization is to reduce the Head Injury Criterion,

$$\text{HIC-d} = 166.4 + 0.75466 * \text{HIC} \tag{12}$$

(as measured at the FMH's center of gravity) by modifying the trim design. The five design variables used are the trim thickness, rib height and thickness, number of ribs and rib span (distance between the first and last rib). Note that the inclusion of the number of ribs as a design variable makes this an integer-based optimization problem. Adaptive meshing is incorporated in the parameterization of the mesh through the TrueGrid (XYZ, 2000) preprocessor to ensure good mesh quality for all possible designs. Note that this is an unconstrained minimization problem as no limits are placed on e.g. the intrusion into the trim or on the mass of the trim.

Results and discussion

Hock and Schittkowski problems

The results for the 38 problems are summarized in Tables I and II. The results obtained using Powell's Sequential Quadratic Programming (SQP) method as reported by Hock and Schittkowski are given in Table I, while the results for the SRSM and SLP method are given in Table II. *n* is the number of design variables.

Convergence is defined in terms of the objective function, with the number of iterations required for 1% and 0.01% convergence given in Tables I and II. The error on the objective is defined as

$$f_{err} = \frac{\left| f_{act} - f^* \right|}{1 + \left| f_{act} \right|} \times 100\% \tag{13}$$

where f_{act} is the exact objective function value (Hock, 1981) and f^* is the computed optimum.

For the SQP results, only final convergence values are available, and the iterations to this final value and the error are given. Note that for each iteration, the objective function, constraint function(s) (if present) and their gradients must be evaluated. SRSM employs $1.5(n + 1) + 1$ *D*-optimal design points for each iteration, while the SLP method uses a small finite-difference step size (10^{-6}), therefore requiring only $n + 1$ evaluations for the numerical gradient. For all the problems, unless otherwise indicated, the original subregion is 25% of the design space in each variable. No problems other than those reported here were attempted.

The result of the twelve-corner polytope problem of Svanberg (1995, 1999) is also given in Tables I and II. Svanberg listed the optimum as 280, found in about 150 iterations (50 outer with about 3 inner iterations each) to an accuracy of 10^{-6} using the Method of Moving Asymptotes (MMA) algorithm. Astonishingly, the SLP method finds this optimum to within 10^{-2} in 7 and to within 10^{-4} in 8 iterations.

Summary of tabled results:

- The SQP method fails to find a local minimum in 2 of the 37 problems it was tested on.
- The SRSM method fails to find a local minimum in 5 of the 38 problems with modification to the default heuristics only required once for convergence.
- The SLP method fails to find a local minimum in 4 of the 38 problems.

For three of the problems where SQP and SLP failed to converge to the global optimum (Problems 16, 33, and 63), SRSM performed better. E.g. for Problem 16, SRSM found the optimum in 80 iterations, but only through the alteration of $\gamma_{\rm pan}$ in Equation 8 from the default value of 1.0 to 1.2. This is the only such amendment in this study. The SQP method, on the other hand, found the global optimum in Problems 13 and 20, while SRSM and SLP converged to local minima. Both SQP and SLP found the correct optimum in Problem 15, while SRSM converged to a local minimum. It should be emphasized that the results presented are for a single starting design for each problem, and that the ability of some of the algorithms to find the global optimum whilst others found local optima, is based on chance.

Small car crash problem

The starting design and optimum design values of the small car crash problem are shown in Table III together with the bounds on the design variables. Note that the initial design is infeasible due to the violation of the intrusion constraint.

The optimization history for the small car crash problem is shown in Figure 6 for the objective (HIC) and in Figure 7 for the design variables (t_{hood} and t_{bumper}). The correct minimal HIC-value is approximately 106 with zero violation of the intrusion. The effect of the only parameter that the user must select in SRSM, the range of the initial subregion, is also shown in Figure 6. It can be seen that the initial subregion size has an effect on the initial convergence, but that the heuristics of the algorithm removes the influence of this parameter by the 8th iteration, making it robust to this selection for this example.

The effect of the initial range is more pronounced on the history of the design variables (see Figure 7), as the initial linear response approximation is less accurate for the larger ranges (4 and 5mm). As soon as the zooming parameter is activated, the subregion becomes smaller and the approximations more accurate, resulting in reduced oscillation in the design variable values as convergence is approached.

Figure 8 shows that simulation results and the response surface predictions converge by about the fourth iteration for an initial range of 2.0mm. The comparison is interesting because it shows the response surface accuracy and the degree of noise present in the problem.

Material identification problem

The starting design and optimum design values of the material identification problem are shown in Table IV together with the bounds on the design variables.

The optimization history for the design variables is given in Figure 9 as a function of the initial range. It can be seen that although SRSM is sensitive to this parameter, the algorithm is robust. The stable convergence rate can also be viewed in the objective function (least-squares error) history plot in Figure 10.

Head impact problem

The starting design and optimum design values of the head impact problem are shown in Table V together with the bounds on the design variables.

The objective function history is given in Figure 11. The solid line represents the result interpolated from the response surface while the solid squares indicate the simulated objective at the current design. As the optimization progresses, the difference between these two diminishes due to the improvement in the approximation. Figure 11 also demonstrates that although SRSM does not include an integer optimization method, it succeeds in the present example in converging to a solution likely to be near an optimum. This example seems to have more noise than the crash problem.

The initial and optimum designs are compared side by side in Figure 12. The reduction in HIC-d is due to a more gradual deceleration of the Free Motion Headform (FMH) upon impact. Figure 13 illustrates how the optimum design cushions the impact by removing the peak in the acceleration curve.

Conclusions

A Successive Response Surface Method (SRSM), specifically tailored for simulation-based optimization, was presented in this paper and tested on a variety of test cases.

The following conclusions can be drawn:

1. The SRSM method performed surprisingly well on the analytical test problems, even though it only used linear approximations. Convergence was in general slower than for SQP, but the contracting subregion helped the algorithm to move into close proximity of the optimum. In general, progress to the region of the optimum is rapid, followed by an expected slow convergence to a higher accuracy.
2. In the engineering test cases, the SRSM method exhibited stable convergence characteristics and the robustness of the method proved to be insensitive to the selection of the initial subregion size.

3. In the final test case, SRSM was able to successfully include an integer variable in the optimization process. Although the success rate of this application is not evident, it is an indication that SRSM is able to deal with the noise induced by approximating a continuous variable with an integer. A more rigorous approach would be to conduct a discrete optimization of the approximate subproblem.
4. An SLP algorithm based on the same domain reduction scheme as SRSM proved to be successful for coarse convergence although it is expected to be successful only for smooth analytical problems.

Finally, the results in this paper demonstrate that, when considering coarse convergence properties, the performance of the Successive Response Surface Method does not differ dramatically from other, more established algorithms such as SQP. While the failure of numerical gradient-based methods such as SQP is well documented for noisy problems, it has been shown that SRSM has the potential of obtaining, with a reasonable degree of accuracy and without experimentation with user-selected parameters, converged optimization solutions to these problems. This makes the algorithm ideal for multidisciplinary optimization problems in which multi-point approximations are suitably constructed for noisy functions (e.g. from crash simulations) and analytical gradients are available for smooth functions (e.g. modal frequencies).

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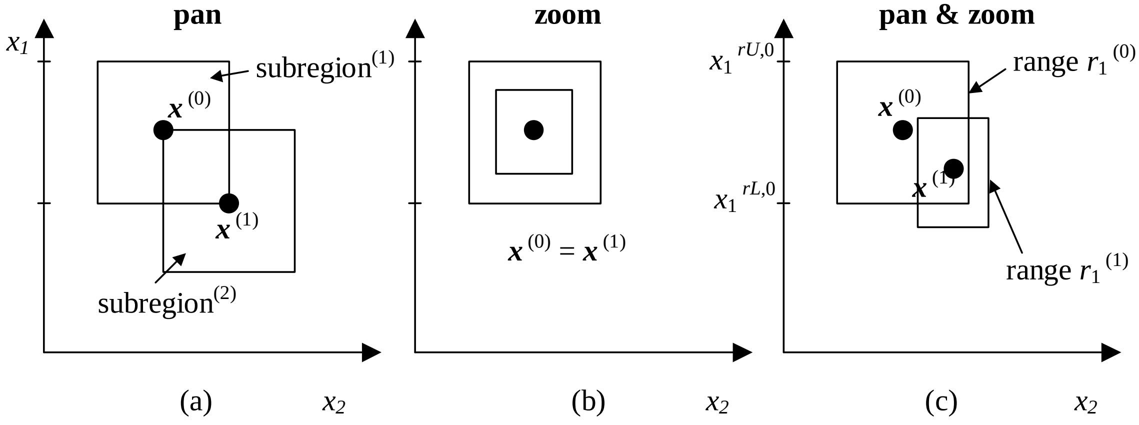

Figure 1 – Adaptation of subregion in SRSM: (a) pure panning, (b) pure zooming and (c) a combination of panning and zooming

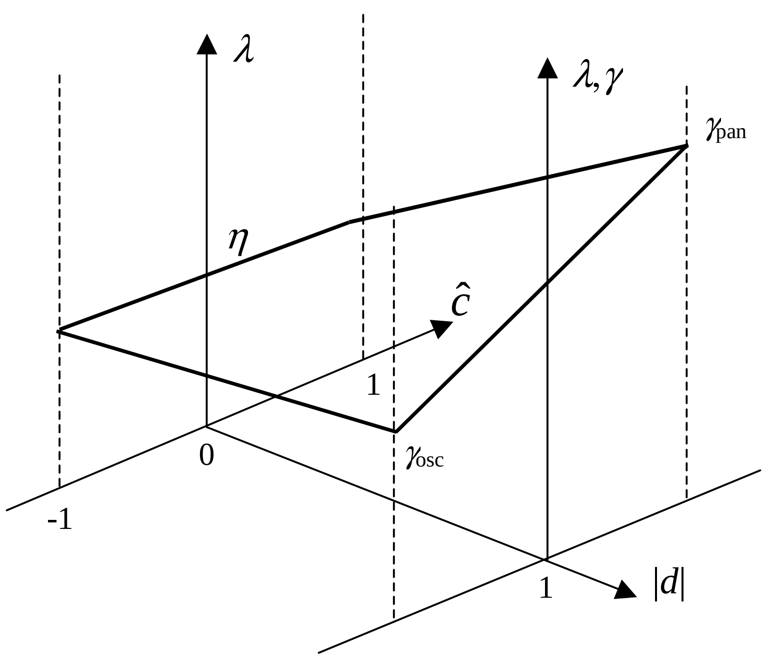

Figure 2 – The sub-region contraction rate λ as a function of the oscillation indicator $\hat{c}$ and the absolute move distance $|d|$

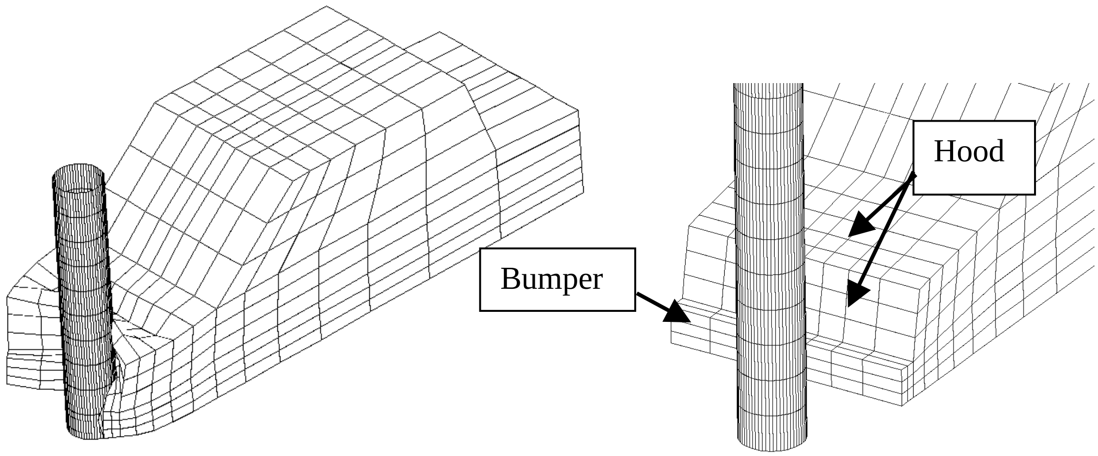

Figure 3 – Small car crash: geometry of deformed (50ms) and undeformed shape

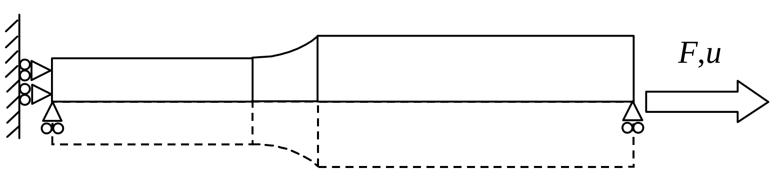

Figure 4 – Quarter symmetric model of test specimen

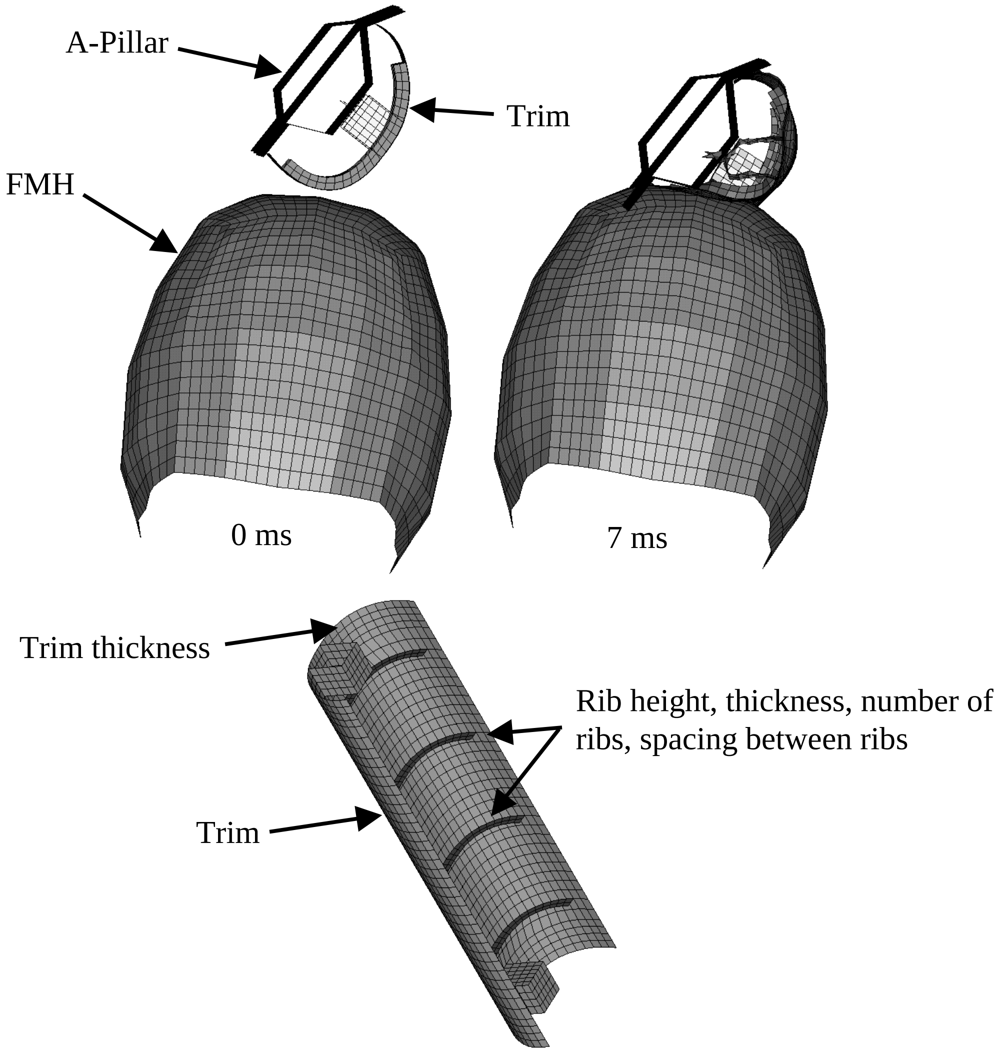

Figure 5 – Head impact problem: Design variables and trim deformation due to impact of FMH

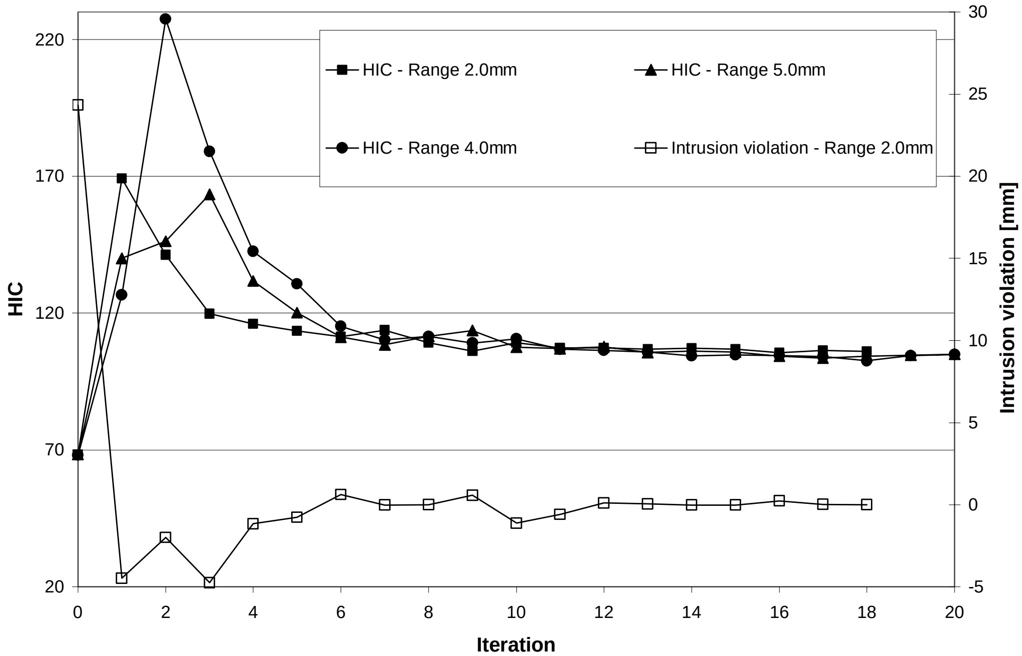

Figure 6 – Small car crash: Optimization history of HIC and intrusion

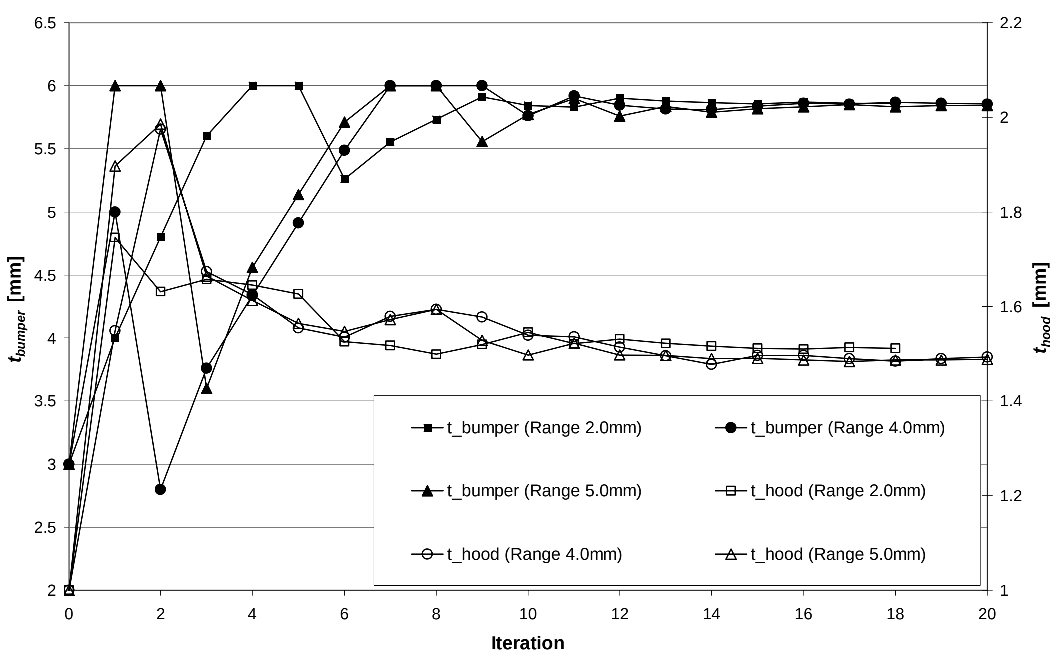

Figure 7 – Small car crash: Optimization history of t_{hood} and t_{bumper}

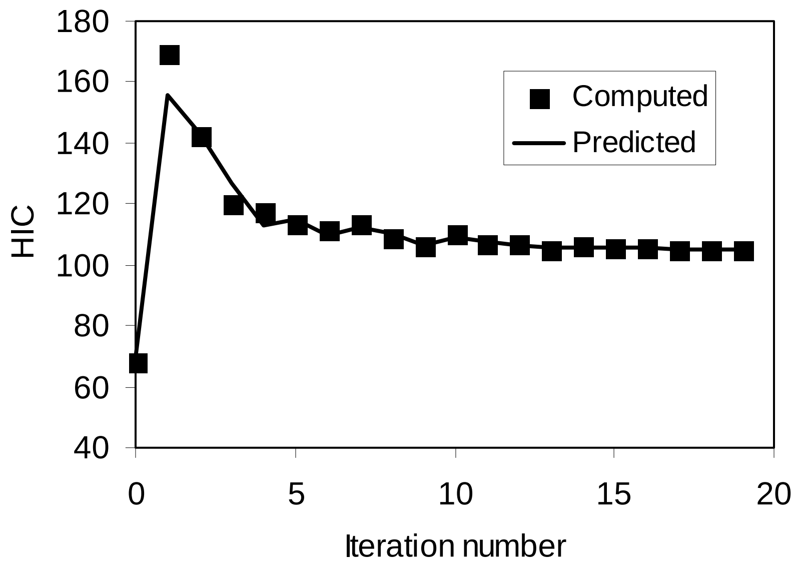

Figure 8 – Small car crash: Optimization history of HIC — simulation results (dots) and response surface results (line). Initial range = 2.0mm.

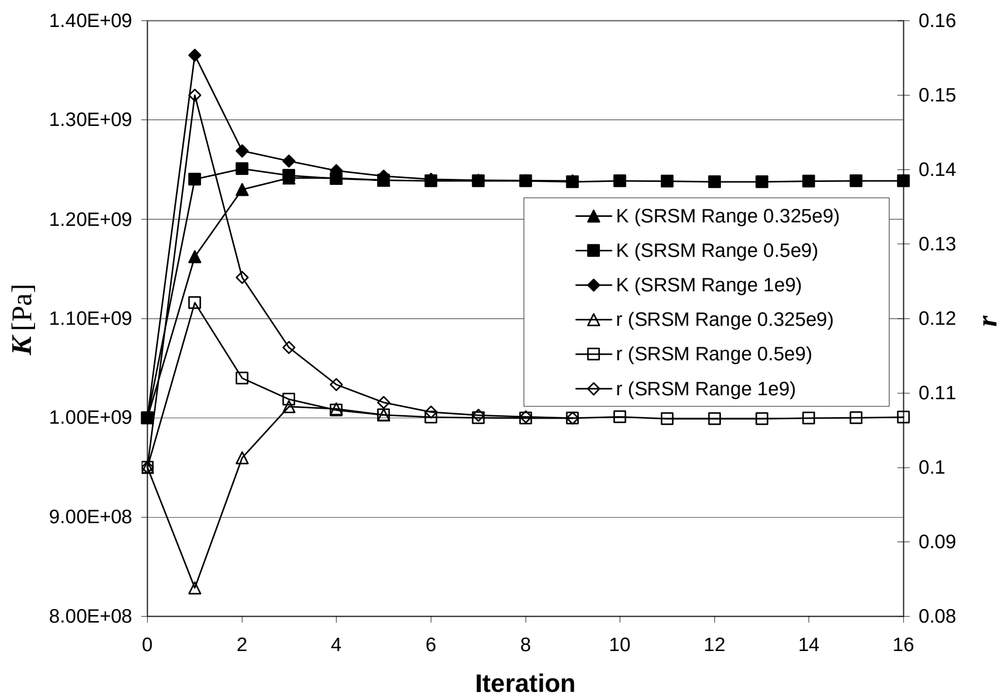

Figure 9 – Material Identification: Optimization history of design variables as a function of initial range on K (range on r = 0.05)

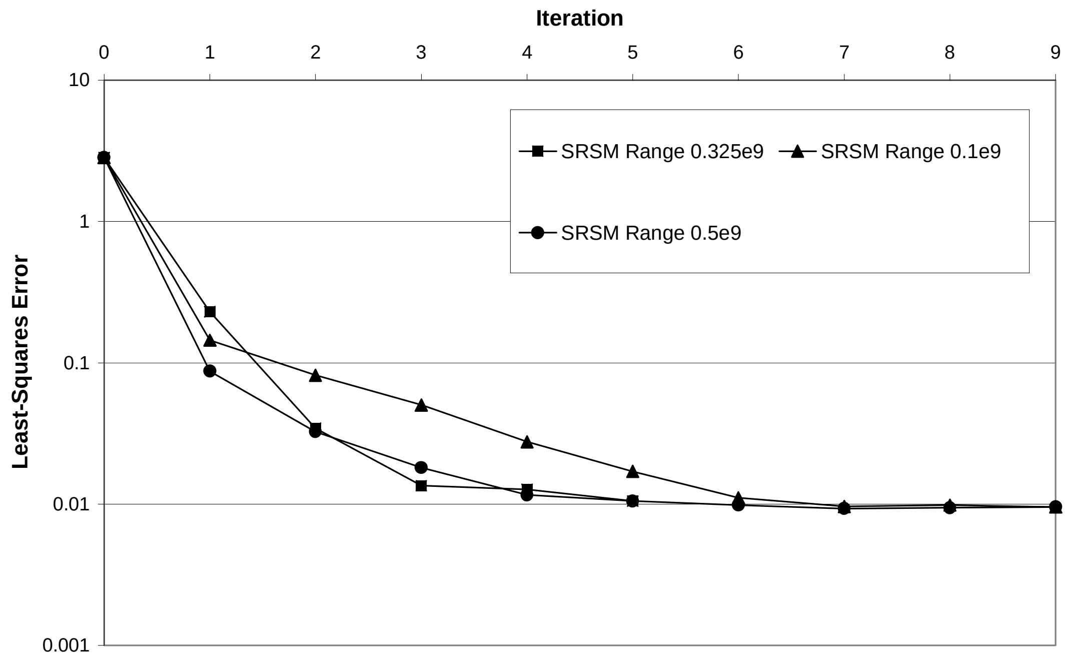

Figure 10 – Material Identification: Optimization history of least-squares error for SRSM

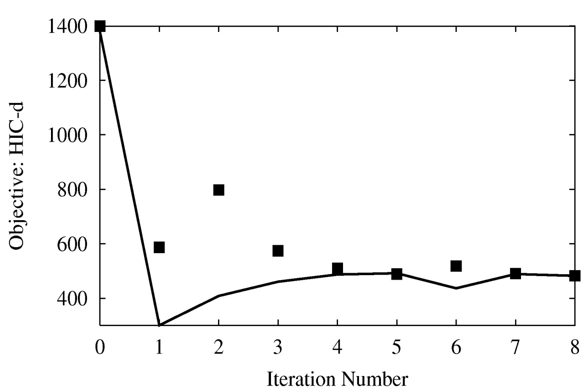

Figure 11 – Objective function history for head impact problem. The squares represent simulation results and the solid line the response surface interpolations at the predicted optima after each iteration.

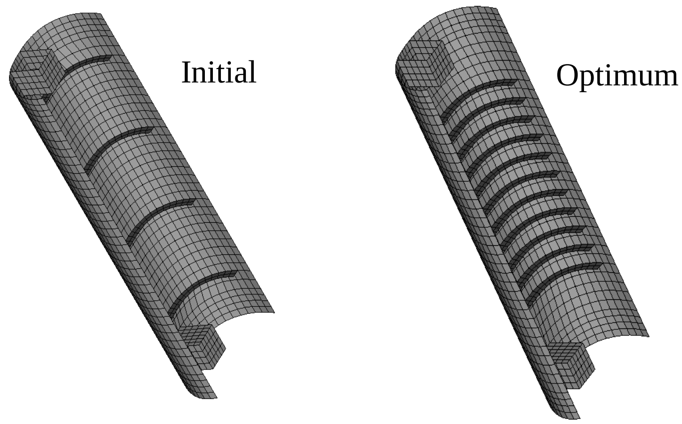

Figure 12 – Head impact problem: Initial and optimum trim designs

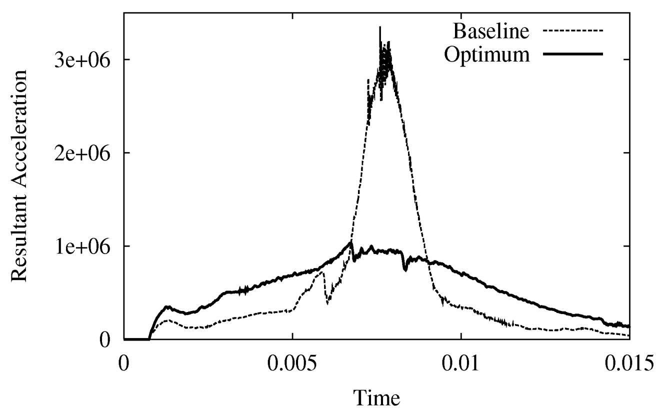

Figure 13 – Head impact problem: Initial (baseline) and optimum FMH acceleration versus time

Problem #	*n*	f_{act}	SQP		
			f^*	Niter	f_{err}
2	2	0.0504	28.4	-	-
10	2	-1	-1	12	5e-8
12	2	-30	-30	12	1e-8
13	2	1	1	45	5e-8
14	2	1.39	1.39	6	8e-9
15	2	307	307	5	1e-8
16	2	0.25	23.1$^+$	-	-
17	2	1	1	12	1e-8
20	2	38.2	38.2	20	5e-9
22	2	1	1	9	1e-8
23	2	9	9	7	1e-8
24	2	-1	-1	5	1e-8
26	3	0	0	19	4e-8
27	3	0.04	0. 04	25	2e-8
28	3	0	0	5	3e-21
29	3	-22.6	-22.6	13	9e-11
30	3	1	1	14	1e-8
31	3	6	6	10	1e-8
32	3	1	1	3	1e-8
33	3	-4.59	-4$^+$	-	-
36	3	-3300	-3300	4	1e-8
45	5	1	1	8	1e-8
52	5	5.33	5.33	8	6e-9
56	7	-3.46	-3.46	11	1e-8
60	3	0.0326	0.0326	9	3e-8
61	3	-144	-144	10	2e-8
63	3	952$^¥$	962$^+$	-	-
65	3	0.954	2.8	-	-
71	4	17.0	17.0	5	2e-8
72	4	728	728	35	1e-8
76	4	-4.68	-4.68	6	3e-9
78	5	-2.92	-2.92	9	3e-9
80	5	0.0539	0.0539	7	8e-10
81	5	0.0539	0.0539	8	2e-9
104	8	3.95	3.95	19	8e-9
106	8	7050	7050	44	1e-5
108	9	-0.866	-0.697$^+$	-	-
12-corner polytope$^\#$	21	280	280	150	1e-6

Table I – Hock and Schittkowski problems (SQP): number of iterations Niter corresponding to objective f^* (error f_{err} and known optimum f_{act})

¥ SRSM found a lower optimum than that listed in Hock & Schittkowski (1981)

\+ Converged to local optimum   # Obtained by MMA (Svanberg 1995, 1999), not SQP

Problem #	n	f_{act}	SRSM			SLP		
			f^*	Niter (1%)	Niter (0.01%)	f^*	Niter (1%)	Niter (0.01%)
2	2	0.0504	6.55	-	-	0.524	-	-
10	2	-1	-1	13	18	-1	24	27
12	2	-30	-30	5	11	-30	5	7
13	2	1	0.76	-	-	0.781	-	-
14	2	1.39	1.39	9	13	1.39	4	5
15	2	307	360^+	-	-	306	5	-
16	2	0.25	$0.25^\$$	68	79	23.1^+	-	-
17	2	1	1	8	11	1	6	6
20	2	38.2	40.2^+	-	-	40.2^+	-	-
22	2	1	1	8	12	1	5	5
23	2	9	9	13	18	9	1	2
24	2	-1	-1	2	2	-1	2	2
26	3	0	0	15	22	0	9	11
27	3	0.04	0.079	-	-	0.072	-	-
28	3	0	0	10	14	0	11	12
29	3	-22.6	-22.6	7	16	-22.6	5	9
30	3	1	1	9	10	1	9	12
31	3	6	6	8	15	6	8	11
32	3	1	1	1	1	1	2	2
33	3	-4.59	-4.59	4	9	-4^+	-	-
36	3	-3300	-3300	5	5	-3300	5	5
45	5	1	1	6	6	1	6	6
52	5	5.33	5.33	9	15	5.33	6	11
56	7	-3.46	-3.46	15	25	-3.46	10	12
60	3	0.0326	0.0326	11	15	0.0326	11	23
61	3	-144	-144	6	11	-144	4	6
63	3	$952^¥$	952	2	8	962^+	-	-
65	3	0.954	0.954	18	22	0.954	14	16
71	4	17.0	17.0	4	10	17.0	2	5
72	4	728	728	34	53	820^+	-	-
76	4	-4.68	-4.68	5	13	-4.68	3	8
78	5	-2.92	-2.92	20	28	-2.92	9	12
80	5	0.0539	0.0539	7	11	0.0539	1	6
81	5	0.0539	0.079	-	-	0.0539	4	6
104	8	3.95	3.95	8	14	3.95	8	18
106	8	7050	7050	8	13	7049	4	5
108	9	-0.866	-0.866	27	32	-0.675^+	-	-
12-corner polytope	21	280	279	7	-	280	7	8

Table II – Hock and Schittkowski problems: number of iterations (Niter) corresponding to objective f^* (SRSM and SLP)

$ $\gamma_{pan} = 1.2$ + Converged to local optimum

¥ SRSM found a lower optimum than that listed in Hock & Schittkowski (1981)

	Minimum	Initial	Maximum	Optimum
t_{hood} [mm]	1	1	6	1.51
t_{bumper} [mm]	1	3	6	5.85
HIC		68.33		106.78
Intrusion violation [mm]		24.34		0

Table III – Small car crash: Design variable upper and lower bounds; initial and optimum values of objective, design variables and constraint

	Minimum	Initial	Maximum	Optimum
K [GPa]	0.7	1	2	1.23865
r [-]	0.01	0.1	0.2	0.106726

Table IV – Material Identification: Design variable upper and lower bounds; initial and optimum values of design variables

	Minimum	Initial	Maximum	Optimum
Trim thickness [mm]		2		2.9
Rib thickness [mm]	0.8	1	1.8	0.8
Rib height [mm]	6	6	15	6.5
Number of ribs [-]	4	4	16	11
Rib span [mm]	130	180	180	140
HIC-d		1400		482

Table V – Head Impact Problem: Design variable upper and lower bounds; initial and optimum design values of objective and design variables